Does The Internet Still Demonstrate Fractal Nature?

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***Abstract*— The self-similar nature of bursty Internet traffic has been investigated for the last decade. A first generation of papers, approximately from 1994 to 2004, argued that the traditionally used Poisson models oversimplified the characteristics of network traffic and were not appropriate for modeling bursty, local-area, and wide-area network traffic. Since 2004, a second generation of papers has challenged the suitability of these results in networks of the new century and has claimed that the traditional Poisson-based and other models are still more appropriate for characterizing today's Internet traffic. A possible explanation was that as the speed and amount of Internet traffic grow spectacularly, any irregularity of the network traffic, such as self-similarity, might cancel out as a consequence of high-speed optical connections, new communications protocols, and the vast number of multiplexed flows. These papers analyzed traffic traces of Internet backbone collected in 2003. In one of our previous papers we applied the theory of smoothly truncated Levy flights and the linear fractal model in examining the variability of Internet traffic from self-similar to Poisson. We demonstrated that the series of interarrival times was still close to a self-similar process, but the burstiness of the packet lengths decreased significantly compared to earlier traces. Since then, new traffic traces have been made public, including ones captured from the Internet backbone in 2008. In this paper we analyze these traffic traces and apply our new analytical methods to illustrate the tendency of Internet traffic burstiness. Ultimately, we attempt to answer the question: Does the Internet still demonstrate fractal nature?**

***Index Terms*— Network traffic, Burstiness, Lévy Flights, Long-range dependence, Fractal modeling.**

I. Introduction

Traffic that is bursty on many or all time scales can be characterized statistically using the concept of self-similarity. Self-similarity is often associated with objects in fractal geometry, that is, objects that appear to look alike regardless of the scale at which they are viewed. In the case of stochastic processes, like time series, self-similarity refers to the process' distribution; when viewed at varying time scales, the process' distribution remains the same. A self-similar time series has noticeable bursts—long periods with extremely high values on all time scales. Characteristics of network traffic, such as interarrival times or length of frames, can be considered as stochastic time series. Therefore, measuring traffic burstiness is the same as characterizing the self-similarity of the corresponding time series.

The self-similarity of network traffic in the last decade was observed in numerous papers, such as [1], [2], [3] and [4]. These measurements of local-area network traffic [5] and wide-area network traffic [6] proved that the widely used Markovian process models could not be used to characterize network traffic. If the traffic was a Markovian process, the traffic's burst length would be smoothed by being averaged over a long time scale, which contradicted with the observations of the traffic characteristics at that time.

Various papers discuss the impact of burstiness on network congestion. Their conclusions are:

- Congested periods can be quite long with losses that are heavily concentrated.
- Linear increases in buffer size do not result in large decreases in packet drop rates.
- A slight increase in the number of active connections can result in a large increase in the packet loss rate.

Results show that packet traffic "spikes" (which cause actual losses) ride on longer-term "ripples," which in turn ride on still longer-term "swells" [[6]].

Many previous works also analyzed the burstiness and the correlation structure of Internet traffic in various time scales in terms of the protocol mechanisms of the TCP, such as time-outs, congestion avoidance, self-clocking, etc. The paper [7] illustrated that short time-scale burstiness is independent of the TCP flow arrival process and showed that in networks with light traffic, correlations across different flows did not have an effect on the short scale burstiness. The same authors illustrated in [8] that a Poisson cluster process could model the aggregate traffic where the packet interarrivals within individual clusters of each flow could be characterized by an overdispersed Gamma distribution. At the same time, the flow volumes showed heavy-tailed properties. Internet traffic was classified in alpha and beta flows in the paper [9]. It was shown that large transfers over high-capacity links, called alpha flows, produced non-Gaussian traffic, while the beta flows—low-volume transmissions—produced Gaussian and long-range dependent traffic. Long sequences of back-to-back packets can cause significant correlations in short time scales.

The majority of the papers focus on the modeling issues of bursty traffic, but there are also an increasing number of papers dealing with controlling self-similar traffic. For instance, the paper [10] presents an algorithm to control self-similar traffic. According to the authors, the algorithm can reduce the burstiness of packet flows at the intermediate routers before forwarding them. Other papers, such as [11] stresses the importance of the accurate and reliable measurement of bursty Internet traffic. The paper [11] discusses significant techniques for performance evaluation and control tools used in traffic engineering.

Some recent papers have taken a different path [12], [13]. The authors of [12] believe that it is time to revisit the Poisson traffic assumption in the Internet backbone. Their position is that traditional Poisson models can be used again to represent the characteristics of the new types of traffic flows in the high-speed backbone of the Internet at the beginning of the decade

978-0-7695-3552-4/09 $25.00 © 2009 IEEE
DOI 10.1109/ICN.2009.8

[14], [15]. They argue that as the amount of Internet traffic grows dramatically, the huge number of different multiplexed flows can moderate and eventually eliminate irregularities of the network traffic, such as burstiness. The paper reports the analysis of traces of the Internet backbone from 2003. The authors found that packet arrivals appeared Poisson at sub-second time scales, the traffic appeared nonstationary at multi-second time scales, and the traffic exhibited long-range dependence at scales of seconds and above.

In one of our previous papers [13] we applied the theory of Smoothly Truncated Levy Flights and the linear fractal model in examining the variability of Internet traffic from self-similar to Poisson. The paper demonstrated that the series of interarrival times of 2003 traces of the Internet backbone was still close to a self-similar process, but the burstiness of the packet lengths decreased significantly compared to earlier traces. Instead of taking a yes-or-no position, we characterized these traces with three parameters of Lévy Flights and positioned a particular trace somewhere in the space generated by the Poisson and self-similar Lévy processes.

In this paper we analyze traffic traces captured in both directions from an OC192 link of the Internet backbone in 2008. We apply our new analytical methods to illustrate the tendency of the fractal nature of the Internet traffic. We are characterizing the network traffic trace as a time series of the arrival time of the packets. The instruments of our analysis are the so called Truncated Lévy Flights [16].

We show that the burstiness of the interarrival times decreased significantly compared to earlier traces. Furthermore, we found that in many traces the series of the interarrival times was Gamma distributed getting closer to Poisson.

The second section describes the mathematical models applied for the analyses of the traces. The third section discusses the types of traces used in our work. The fourth sections present the results of the application of our models to the data, followed by the conclusion in section five and the Appendix containing the detailed descriptions of the methods used in the paper.

II. Traffic Traces

We analyzed packet traces collected for four hours by CAIDA[1] in May, 2008. The data sets contained anonymized traffic traces from an Internet data collection monitor on an OC192 Internet backbone link (9953.28 Mbps). The Internet data collection monitor was located in Chicago, IL, and was connected to a Tier1 ISP between Chicago, IL and Seattle, WA.

The traffic was captured by two network monitoring cards in both directions. A single card was connected to a single direction of the full-duplex backbone link. The directions were denoted by A (Seattle to Chicago) and B (Chicago to Seattle). Due to the size of the original trace files (Compressed size of direction A is 4.1GB, compressed size of the trace in direction B is 14 GB), we took only smaller, couple of minutes samples from both directions: Trace A was 2GB and Trace B was 3.2GB. These traces were further divided into twelve slices each with one million bytes in lengths ($2^{20} = 1048576$) denoted by A1, ..,A12 and B1,...,B12 respectively.

The capture cards supported a timestamp precision of around 233 picoseconds. Since the card's output file format was not supported by the majority of traffic analysis tools, CAIDA converted the original traces to a format with nanosecond timestamp precision along with the packet lengths for both IPv4 and IPv6 packets separately.

It is noticeable from the size of the traces that direction A had less traffic then direction B. A possible reason of the difference is that many content servers were located at one end of the link. Another interesting observation of the traffic was that based on a smaller sample, only a small portion (~8.6%) of IPv4 addresses was captured as both source and destination IP addresses in packets after merging both directions. This could be the indication that the network traffic in this area of the backbone may have been routed asymmetrically (Email communications with Emile Aben, Data Administrator, CAIDA/SDSC/UCSD).

[1]Support for CAIDA's OC192 Traces is provided by the National Science Foundation, the US Department of Homeland Security, DARPA, Digital Envoy, and CAIDA Members

III. Our Model

A. Smoothly Truncated Lévy Flights

In this section we introduce a model: The Smoothly Truncated Levy Flights (STLFs). The concept of the more general distribution, called tempered stable distribution, is due to Rosiński [18] (see, e.g., [16] and [19] for a partial history of these works). STLFs will be applied below for describing the distribution of the interarrival times of the packet traces. Since the interarrival times are positive, we consider STLF with a totally asymmetric distribution. It is given by the cumulant function (log of the characteristic function)

$$\psi_X(u) = a\Gamma(-\alpha)\left[(\lambda - iu)^\alpha - \lambda^\alpha\right], \tag{1}$$

where $\alpha \in (0, 1)$ and $\lambda, a > 0$. A more general discussion of STLF is given in Appendix. This distribution depends on three parameters: the *index* α, the *truncation* parameter λ, and the *scale* parameter a. These parameters introduced in [13] provide some information about the position of the distribution in the following manner:

Property 1. If α and a are fixed and λ *tends to zero*, then the limit distribution is a totally asymmetric $\alpha-$stable distribution and the corresponding Lévy process is self-similar.

Property 2. If λ and a are fixed and α *tends to zero*, then the limit distribution is Gamma with parameters (a, λ). In particular, if a is 1, then the limit is exponential, therefore the Lévy process is Poisson.

Property 3. If λ and α are fixed, then for small a the distribution is close to the α- stable distribution and for large a the distribution is close to Gaussian.

Both λ and a depend on the scaling of interarrival times X. When α is zero and the parameter a equals one, then the distribution is exponential. Since we are interested in the distance of the traces from being exponential, we choose $a = 1$. Therefore, the distance from the $\alpha-$stable distribution can be measured by the single parameter λ provided α is small.

More precisely, if X is distributed as $STLF_{\alpha}(a, 0, \lambda)$, then the distribution of cX is $STLF_{\alpha}(ac^{\alpha}, 0, \lambda/c)$, where $c > 0$. In case $\alpha = 0$, we get Propety 2 above.

For $m \geq 1$, the cumulants, derived from the cumulant function (1), are given in terms of the parameters α, λ, and a, namely,

$$cum_m(X) = a\lambda^{\alpha-m}\Gamma(m-\alpha). \tag{2}$$

IV. Packet Interarrival Times

We are going to search for a distribution that would be suitable for characterizing the interarrival times by applying the family of Lévy processes. The Poisson process is one of the simplest Lévy processes (see, e.g., [24]) with the main assumption that the increments—the interarrival times— are independent, homogeneous and exponential. Changing the distribution of the increments we obtain a wide variety of Lévy-stable processes as candidates for modeling the interarrival times [25]. The heavy tail of a distribution also implies that the moments do not exist, so these distributions are not appropriate for modeling purposes. Other members of the family of Lévy processes, the Smoothly Truncated Lévy Flights (STLF), have higher order moments. Since they have been successfully applied for finance, biological, and physical phenomena it is reasonable to apply it for traffic analysis as well. Some applications of the STLF are demonstrated in [26], [27], [28], and [16]. The following formula of the cumulants of STLF provides a means for estimating the parameters by the method of moments, i.e., calculating the empirical values from the traffic traces and compare them with the theoretical values above:

$$cum_m(X) = a\lambda^{\alpha-m}\Gamma(m-\alpha),$$

More precisely, for a given trace we calculate the estimated cumulants $\widehat{cum_m}$, $m = 1, 2, \ldots 8$, then we use the least squares method for finding the estimates $\widehat{\alpha}$, $\widehat{\lambda}$, and $\widehat{a}$ (for the details, please see the authors).

We carried out these calculations for the OC192 traces. The Table 1 and Table 2 contain the estimated parameters $\widehat{\alpha}$, $\widehat{\lambda}$, and $\widehat{a}$ for direction A and B respectively. The second column of Table 1 shows that half of the alpha values of the traces in direction A is close to zero, therefore, it follows from property 2 that the limit distribution is close Gamma with parameters ($\widehat{\alpha}$, $\widehat{\lambda}$). Please note that for traces A9, A10, and A12 the alpha values are zero, therefore we consider the distribution being Gamma. The majority of the alpha values in direction B is around 1/2 corresponding to the Inverse Gaussian distribution.

Traces	$\widehat{\alpha}$	$\widehat{\lambda}$	$\widehat{a}$
A1	0.09658	0.04486	1
A2	0.15645	0.08802	1
A3	0.30021	0.12282	1
A4	0.17906	0.09853	1
A5	0.11054	0.05847	1
A6	0.09934	0.04399	1
A7	0.20211	0.11745	1
A8	0.08422	0.03182	1
A9	0	0.16318	0.86673
A10	0.00699	0.15841	0.81487
A11	0.08522	0.02687	1
A12	0	0.16705	0.84965

Table 1. Estimated STLF parameters, direction A.

Traces	$\widehat{\alpha}$	$\widehat{\lambda}$	$\widehat{a}$
B1	0.47701	0.08701	1
B2	0.44731	0.09006	1
B3	0.40701	0.08398	1
B4	0.43144	0.08824	1
B5	0.45038	0.09043	1
B6	0.27388	0.0696	1
B7	0.44736	0.08575	1
B8	0.28557	0.07404	1
B9	0.4586	0.09149	1
B10	0.32728	0.07944	1
B11	0.58562	0.07973	1
B12	0.36027	0.08969	1

Table 2. Estimated STLF parameters, direction B.

Figure 1 corresponds to slice A9 in Table 1 shows the log of estimated cumulants $\widehat{cum_m}$, and the log of cumulants $\widetilde{cum_m}$, $m = 1, 2, \ldots 8$, of the Smoothly Truncated Lévy Flights when the parameters are estimated, i.e.,

$$\widetilde{cum_m}(X) = \widehat{a}\widehat{\lambda}^{\widehat{\alpha}-m}\Gamma(m-\widehat{\alpha}).$$

It is shown in figure 1 that the fitting is very good.

V. Conclusion

In this paper we attempted to answer the question: Does the Internet still have fractal properties? We analyzed traffic traces captured by CAIDA in May 2008 from an OC192, high-speed link of the Internet in both directions, A and B. We applied our model to the traces to illustrate the trend of Internet traffic in terms of self-similarity relative to previous traffic traces. Our model, built on the Truncated Lévy Flights, positioned a particular trace somewhere in the space generated by the Poisson and self-similar Lévy processes. We compared the series of the interarrival times of the OC192 traces in direction A with the interarrival times in direction B. We found that the burstiness of the interarrival times decreased significantly compared to earlier traces, especially in direction A. Furthermore, we found that in many traces the distribution of the interarrival times was Poisson deviating from previous observations. Therefore, in answering our original question, we can conclude that based on the sample traces, the Internet is losing its self-similar nature that was so prevalent for years.

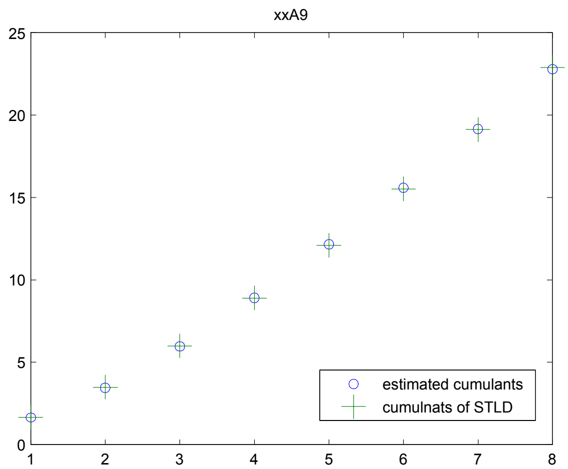

Fig. 1. Comparison of the STLF log-cumulants and estimated ones of the OC192 trace.

Acknowledgment. The authors thank Emile Aben, Data Administrator, CAIDA/SDSC/UCSD for his valuable comments and the data sets he created for this paper.

VI. Appendix

A. STLF

Let us recall that the STLF $X(t)$ is a Lévy process, i.e., a process with homogeneous and independent increments and $X(0) = 0$. The probability distribution of $X = X(1)$ has characteristic function of the form

$$\varphi_X(u) = \exp(\psi_X(u)),$$

where the *cumulant function*

$$\psi_X(u) = a\lambda^\alpha [p\zeta_\alpha(-u/\lambda) + q\zeta_\alpha(u/\lambda)] + iub,$$

and $\lambda > 0,\ a, p, q \geq 0,\ p + q = 1$, b is a real number, and

$$\zeta_\alpha(r) = \begin{cases} \Gamma(-\alpha)\left[(1-ir)^\alpha - 1\right], & \text{for } 0 < \alpha < 1; \\ (1-ir)\log(1-ir) + ir, & \text{for } \alpha = 1; \\ \Gamma(-\alpha)\left[(1-ir)^\alpha - 1 + i\alpha r\right], & \text{for } 1 < \alpha < 2. \end{cases}$$

(See [16] for details.)

Without loss of generality, we only consider the case when the shift parameter $b = 0$. Parameters p and q describe the *skewness* of the probability distributions, and $p = q = 1/2$ yields a symmetric distribution. Parameter λ will be referred to as the *truncation* parameter.

In the case of $0 < \alpha < 1$, the cumulant function is given by the formula

$$\psi_X(u) = a\lambda^\alpha\Gamma(-\alpha)\left[p\left(1 + i\frac{u}{\lambda}\right)^\alpha + q\left(1 - i\frac{u}{\lambda}\right)^\alpha - 1\right], \tag{3}$$

and if $p = 0$, the cumulant function

$$\begin{aligned}\psi_X(u) &= a\lambda^\alpha\Gamma(-\alpha)\left[\left(1 - i\frac{u}{\lambda}\right)^\alpha - 1\right] \\ &= a\Gamma(-\alpha)\left[(\lambda - iu)^\alpha - \lambda^\alpha\right],\end{aligned} \tag{4}$$

describes a distribution totally concentrated on the positive half-line. The distribution of X will be denoted by $STLF_\alpha(a, p, \lambda)$. The index α corresponds to the nontruncated limit when $\lambda = 0$. In this case the distribution of X is the classical Lévy's $\alpha-$stable probability distribution. The scale parameter a tunes the time unit to a, hence the distribution of $X(t)$ is $STLF_\alpha(at, p, \lambda)$.

The role of the truncation parameter λ is obvious in the following particular case. For the one-sided $STLF_\alpha(a, 0, \lambda)$ distribution with $0 < \alpha < 1$, the cumulant function has the form

$$\psi_X(u) = a\lambda^\alpha\Gamma(-\alpha)\left[\left(1 - i\frac{u}{\lambda}\right)^\alpha - 1\right]. \tag{5}$$

As $\lambda \to 0$, the distribution $STLF_\alpha(a, 0, \lambda)$ converges to the $\alpha-$stable distribution $STLF_\alpha(a, 0, 0)$.

For a fixed $\lambda, a > 0$, as $\alpha \to 0$, the distribution $STLF_\alpha$ tends to the Gamma distribution $\Gamma(a, \lambda)$. Indeed, for $0 < \alpha < 1$, the Laplace transform ϕ_λ of $STLF_\alpha(a, 0, \lambda)$ is

$$\phi_\lambda(u) = \exp\left(a\lambda^\alpha\Gamma(-\alpha)\left[(1 + u/\lambda)^\alpha - 1\right]\right),$$

and

$$\begin{aligned}&\lim_{\alpha\to 0}\exp\left(-a\Gamma(1-\alpha)\frac{(\lambda+u)^\alpha - \lambda^\alpha}{\alpha}\right) \\ &\qquad = \exp\left(-a\log(1 + u/\lambda)\right) = (1 + u/\lambda)^{-a},\end{aligned}$$

by the L'Hospital rule.

B. Estimating the parameters of $STLF_\alpha(a, 0, \lambda)$

Take the logarithm of

$$cum_m(X) = a\lambda^{\alpha-m}\Gamma(m-\alpha).$$

We obtain

$$\log cum_m(X) = \log a + (\alpha - m)\log\lambda + \log\Gamma(m-\alpha). \tag{6}$$

Plug the estimated cumulants $\widehat{cum_m}$ into the left side of equation (6), then we have three unknowns a, λ, and α. We obtain

$$\begin{aligned}\widehat{\lambda} &= \frac{\widehat{cum_3}(X)\,\widehat{cum_2}(X)}{\widehat{cum_4}(X)\,\widehat{cum_2}(X) - \left[\widehat{cum_3}(X)\right]^2}, \\ \widehat{\alpha} &= 2 - \frac{\left[\widehat{cum_3}(X)\right]^2}{\widehat{cum_4}(X)\,\widehat{cum_2}(X) - \left[\widehat{cum_3}(X)\right]^2}, \\ \widehat{a} &= \frac{\widehat{cum_2}(X)}{\widehat{\lambda}^{\widehat{\alpha}-2}\Gamma(2-\widehat{\alpha})}.\end{aligned}$$

We obtain more precise estimations for the parameters, if we use these estimates as initial values and refine the estimates using nonlinear least squares, which minimizes

$$\sum_{m=1}^{8}\left[cum_m(X) - a\lambda^{\alpha-m}\Gamma(m-\alpha)\right]^2.$$

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