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GROWTH AND OSCILLATION OF DIFFERENTIAL POLYNOMIALS IN THE UNIT DISC

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Abstract. In this article, we give sufficiently conditions for the solutions and the differential polynomials generated by second-order differential equations to have the same properties of growth and oscillation. Also answer to the question posed by Cao [6] for the second-order linear differential equations in the unit disc.

1. Introduction and main results

The study on value distribution of differential polynomials generated by solutions of a given complex differential equation in the case of complex plane seems to have been started by Bank [1]. Since then a number of authors have been working on the subject. Many authors have investigated the growth and oscillation of the solutions of complex linear differential equations in $\mathbb{C}$, see [2, 4, 7, 10, 13, 17, 18, 19, 21, 25, 28]. In the unit disc, there already exist many results [3, 5, 6, 8, 9, 15, 16, 20, 23, 24, 29], but the study is more difficult than that in the complex plane. Recently, Fenton-Strumia [11] obtained some results of Wiman-Valiron type for power series in the unit disc, and Fenton-Rossi [12] obtained an asymptotic equality of Wiman-Valiron type for the derivatives of analytic functions in the unit disc and applied to ODEs with analytic coefficients.

In this article, we assume that the reader is familiar with the fundamental results and the standard notation of the Nevanlinna's theory on the complex plane and in the unit disc $D = \{z : |z| < 1\}$, see [14, 18, 22, 24, 26, 27]. In addition, we will use $\lambda(f)$ $(\lambda_2(f))$ and $\overline{\lambda}(f)$ $(\overline{\lambda}_2(f))$ to denote respectively the exponents (hyper-exponents) of convergence of the zero-sequence and the sequence of distinct zeros of a meromorphic function f, $\rho(f)$ to denote the order and $\rho_2(f)$ to denote the hyper-order of f. See [9, 15, 20, 24] for notation and definitions.

Definition 1.1. The type of a meromorphic function f in D with order $0 < \rho(f) < \infty$ is defined by

$$\tau(f) = \limsup_{r \to 1^-} (1-r)^{\rho(f)} T(r, f).$$

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 Consider the linear differential equation

$$f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_1(z)f' + A_0(z)f = 0, \quad (1.1)$$

where $A_0, A_1, ..., A_{k-1}$ are analytic functions in D, and k is an integer , $k \geq 1$.

Theorem 1 . 2 ([5]) . *Let* $A_0(z), ..., A_{k-1}(z)$, *the coefficients of* (1 . 1) , *be analytic in* D. *If* $\max\{\rho(A_j) : j = 1, ..., k-1\} < \rho(A_0)$, *then* $\rho(A_0) \leqslant \rho2(f) \leqslant \alpha_M$ *for al l s o lutio ns* $f equivalence-negationslash 0$ *of* (1 . 1) , *where* $\alpha_M = \max\{\rho M(A_j) : j = 0, ..., k-1\}$.

Recall that the order of an analytic function f in D is defined by

$$\rho M(f) = \limsup_{r \to 1-} \frac{\log^+ \log^+ M(r,f)}{\log \frac{1}{1-r}},$$

where $M(r,f) = \max_{|z|=r} | f(z) |$. The following two st atements hold [24 , p . 205] .

(a) If f is an analytic function in D, then $\rho(f) \leqslant \rho M(f) \leqslant \rho(f) + 1$.

(b) There exist analytic functions f in D which satisfy $\rho M(f) \neq \rho(f)$. For example , let $\mu > 1$ be a constant , and set

$$\psi(z) = \exp\{(1-z)^{-\mu}\},$$

where we choose the principal branch of the logarithm . Then $\rho(\psi) = \mu - 1$

$$\text{and} \rho M(\psi) = \mu, \text{see[9]}.$$

In contrast , the possibility that o ccurs in (b) cannot o ccur in the whole plane $\mathbb{C}$, because if $\rho(f)$ and $\rho M(f)$ denote the order of an entire function f in the plane $\mathbb{C}$ (defined by the Nevanlinna characteristic and the maximum modulus , respectively) , then it is well know that $\rho(f) = \rho M(f)$.

Theorem 1 . 3 ([5]) . *Under the hypotheses of Theorem 1 . 2 , if* $\rho2(A_j) < \infty$, $(j = 0, ..., k-1)$, *then every s o lution* $f \not\equiv 0$ *of* (1 . 1) *satisfies* $\bar{\lambda}_2(f - z) = \rho2(f)$.

Consider a linear differential equation of the form

$$f'' + A_1(z)f' + A_0(z)f = F, \quad (1.2)$$

where $A_1(z), A_0(z) equivalence-negationslash 0, F(z)$ are analytic functions in the unit disc $D = \{z :| z |< 1\}$. It is well - known that all solutions of equation (1 . 2) are analytic functions in D and that there are exactly two linearly independent solutions of (1 . 2) ; see [1 5] .

Many important results have been obtained on the fixed points of general tran - scendental meromorphic functions for almost four decades , see [28] . However , there are few studies on the fixed points of solutions of differential equations , specially in the unit disc . Chen [7] studied the problem on the fixed points and hyper - order of solutions of second order linear differential equations with entire coefficients . After that , there were some results which improve those of Chen , see [2 , 1 0 , 1 9 , 2 1 , 25] . It is natural to ask what can be said about similar situations in the unit disc D. Re - cently , Cao [6] investigated the fixed points of solutions of linear complex differential equations in the unit disc .

The main purpose of this article is to give sufficiently conditions for the solutions and the differential polynomials generated by the second order linear differential equa- tion (1 . 2) to have the same properties of the growth and oscillation . Also , we answer to the following question posed by Cao [6] :

How about the fixed points and iterated order of differential poly - nomial generated by solutions of linear differential equations in the unit disc ?

 Before we state our results , we denote

$$\alpha_0 = d_0 - d_2A_0, \quad \beta 0 = d_2A_0A_1 - (d_2A_0)' - d_1A_0 + d_0', \tag{1.3}$$
$$\alpha_1 = d_1 - d_2A_1, \quad \beta 1 = d_2A_1^2 - (d_2A_1)' - d_1A_1 - d_2A_0 + d_0 + d_1', \tag{1.4}$$
$$h = \alpha_1\beta 0 - \alpha_0\beta 1, \tag{1.5}$$
$$\psi(z) = \frac{\alpha_1(\varphi' - (d_2F)' - \alpha_1F) - \beta 1(\varphi - d_2F)}{h}, \tag{1.6}$$

where $A_0, A_1, d_0, d_1, d_2, \varphi$ and F are analytic functions in the unit disc $D = \{z : | z |< 1\}$ with finite order .

Theorem 1 . 4 . *Let $A_1(z), A_0(z) equivalence - negationslash 0$ and F be analytic functions in D, of finite order . Let d_0, d_1, d_2 be analytic functions in D that are not al l equal to zero with $\rho(d_j) < \infty$ $(j = 0, 1, 2)$ such that $h equivalence - negationslash\ 0$, where h is defined by (1 . 5) . If f is an infinite order s o lutio n of (1 . 2) with $\rho 2(f) = \rho$, then the differential polynomial*

$$g_f = d_2f'' + d_1f' + d_0f satisfies$$
$$\rho(g_f) = \rho(f) = \infty, \quad \rho 2(g_f) = \rho 2(f) = \rho. \quad (1.7)$$

Theorem 1 . 5 . *Let $A_1(z), A_0(z) equivalence - negationslash 0$ and F be analytic functions in D of finite order . Let $d_0(z), d_1(z), d_2(z)$ be analytic functions in D which are not al l equal to zero with $\rho(d_j) < \infty$ $(j = 0, 1, 2)$ such that $h equivalence - negationslash 0$, and le t $\varphi(z)$ be an analytic function in D with finite order such that $\psi(z)$ is not a s o lution of (1 . 2) . If f is an infinite order s o lutio n of (1 . 2) with $\rho 2(f) = \rho$, then the differential polynomial*

$$g_f = d_2f'' + d_1f' + d_0f satisfies$$
$$\text{-}\lambda_{(g_f} - \varphi) = \lambda(g_f - \varphi) = \rho(g_f) = \rho(f) = \infty, \quad (1.8)$$
$$\text{-}\lambda_{2(g_f} - \varphi) = \lambda_2(g_f - \varphi) = \rho 2(g_f) = \rho 2(f) = \rho. \quad (1.9)$$

Remark 1 . 6 . In Theorem 1 . 5 , if we do not have the condition $\psi(z)$ is not a solution of (1 . 2) , then the conclusions of Theorem 1 . 5 does not hold . For example ,
the functions $f1(z) = 1 - z$ and $f2(z) = (1 - z) \exp (\exp \frac{1}{1-z})$ are linearly independent solutions of the equation

$$f'' + A_1(z)f' + A_0(z)f = 0, \tag{1.10}$$

where

$$A_0(z) = -\frac{\exp \frac{1}{1-z}}{(1-z)^3} - \frac{1}{(1-z)^3}, \quad A_1(z) = -\frac{\exp \frac{1}{1-z}}{(1-z)^2} - \frac{1}{(1-z)^2}.$$

Clearly $f = f1 + f2$ is a solution of (1 . 1 0) . Set $d_2 = d_1 \equiv 0$ and $d_0 = \frac{1}{1-z}$. Then $g_f = d_0f$, $h = -d_0^2$ and $\psi(z) = \frac{\varphi}{d_0}$. If we take $\varphi = d_0f1$, then $\psi(z) = f1$ is a solution of (1 . 1 0) and we have

$$\lambda(g_f - \varphi) = \lambda(d_0f - d_0f1) = \lambda(d_0f2) = \lambda(\exp(\exp \frac{1}{1-z})) = 0.$$

On the other hand ,

$$\rho(g_f) = \rho(d_0 f) = \rho(d_0 f1 + d_0 f2) = \rho(1 + \exp(\exp \frac{1}{1-z})) = \infty.$$

Theorem 1 . 7 . *Let* $A_1(z), A_0(z) \not\equiv 0$ *and* F *be finite order analytic functions in* D *such that all s o lutions of (1 . 2) are of infinite order . Let* $d_0(z), d_1(z), d_2(z)$ *be analytic functions in* D *which are not al l equal to zero with* $\rho(d_j) < \infty (j = 0, 1, 2)$ *such that* $hequivalence - negationslash 0,$ *and le t* $\varphi(z)$ *be an analytic function in* D *with finite o rder . If* *f is a s o lution of (1 . 2) with* $\rho 2(f) = \rho$, *th en th e differential polynomial* $g_f =$

$$d_2 f'' + d_1 f' + d_0 f satisfies (1.8) and (1.9).$$

Remark 1 . 8 . In Theorems 1 . 4 , 1 . 5 , 1 . 7 , if we do not have the condition $h \not\equiv 0$, then the differential polynomial can be of finite order . For example , if $d_2(z) equivalence - negationslash 0$, is a finite order analytic function in D and $d_0(z) = A_0(z)d_2(z), d_1(z) = A_1(z)d_2(z)$, then $h \equiv 0$ and $g_f = F(z)d_2(z)$ is of finite order .

In the following we give an application of the above results .

Corollary 1 . 9 . *Let* $A_0(z)$, $A_1(z), d_0, d_1, d_2$ *be analytic functions in* D *such that*

$$\max\{\rho(A_1), \rho(d_j)(j = 0, 1, 2)\} < \rho(A_0) = \rho(0 < \rho < \infty), \tau(A_0) = \tau(0 < \tau < \infty),$$

and le t $\varphi equivalence - negationslash 0$ *be an analytic function in* D *with* $\rho(\varphi) < \infty$. *If* $f equivalence - negationslash 0$ *is a s o lution of equation (1 . 1 0) , then the differential polynomial* $g_f = d_2 f'' + d_1 f' + d_0 f$ *satisfies*

$$\lambda(g_f - \varphi) = \overline{\lambda(g_f - \varphi)} = \rho(g_f) = \rho(f) = \infty, \quad (1.11)$$

$$\alpha_m \geqslant \text{-}\lambda_{2(g_f} - \varphi) = \lambda_2(g_f - \varphi) = \rho 2(g_f) = \rho 2(f) \geqslant \rho(A_0), \quad (1.12)$$

$$where \alpha_M = \max\{\rho M(A_j) : j = 0, 1\}.$$

Remark 1 . 10 . The special case $\varphi(z) = z$ in the above theorems reduces to the fixed points of the differential polynomial g_f.

2 . Auxiliary Lemmas

Lemma 2 . 1 ([5]) **.** *Let* $f(z)$ *be a meromorphic s o lution of the equation*

$$L(f) = f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_0(z)f = F(z), \quad (2.1)$$

where k *is an positive integer* $, A_0, ..., A_{k-1}, F \not\equiv 0$ *are meromorphic functions in* D *such that* $\max\{\rho i(F), \rho i(A_j)(j = 0, ..., k-1)\} < \rho i(f), (i = 1, 2)$. *Then ,*

$$\text{-}\lambda_{i(f)} = \lambda_i(f) = \rho i(f) \quad (i = 1, 2). \quad (2.2)$$

Using the properties of the order of growth see [3 , Proposition 1 . 1] and the
definition of the type , we easily obtain the following result which we omit the proof .

Lemma 2 . 2 . *Let* f *and* g *be meromorphic functions in* D *such that* $0 < \rho(f)$, $\rho(g) < \infty$ *and* $0 < \tau(f), \tau(g) < \infty$. *Then the following two s tatements hold :*

(i) *If* $\rho(f) > \rho(g)$, *then*

$$\tau(f + g) = \tau(fg) = \tau(f). \quad (2.3)$$

(i i) *If* $\rho(f) = \rho(g)$ *and* $\tau(f) > \tau(g)$, *then*

$$\rho(f + g) = \rho(fg) = \rho(f) = \rho(g). \quad (2.4)$$

Lemma 2 . 3 . *Let* $A_0(z)$, $A_1(z)$, d_0, d_1, d_2 *be analytic functions in* D *such that*

$$\max\{\rho(A_1), \rho(d_j), (j=0,1,2)\} < \rho(A_0) = \rho(0 < \rho < \infty), \tau(A_0) = \tau(0 < \tau < \infty).$$

Then $h \not\equiv 0$, *where* h *is given by* (1 . 5) . *Proof .* First we suppose that $d_2(z) equivalence - negationslash 0$. Set

$$h = \alpha^{1\ \beta 0}_{-(d_0} -_- \alpha_0 d^{\beta 1\ \ =(d_1}_{2^{A}0)(d_{2^{A^2}_{1}}} -_- d_2(^{A_1)(d_2 A 0\ A_1\ -}_{d_2 A_1)'\ \ -d_1 A 1} - (d^{A_0)'}_{2 d_2}\ A_0 + - d_1 d^{A_0 +}_{0+d\ '1)}.d_0') \tag{2.5}$$

 Now check all the terms of h. Since the term $d_2^2 A_1^2 A_0$ is eliminated , by (2 . 5) we can write

$$\begin{aligned} h = -d_2^2 A_0^2 - d_0 d_2 A_1^2 + (d_1' d_2 + 2d_0 d_2 - d_2' d_1 - d_1^2) A_0 \\ +(d_2' d_0 - d_2 d_0' + d_0 d_1) A_1 + d_1 d_2 A_0 A_1 - d_1 d_2 A_0' \\ +d_0 d_2 A_1' + d_2^2 A_0' A_1 - d_2^2 A_0 A_1' + d_0' d_1 - d_0 d_1' - d_0^2. \end{aligned} \tag{2.6}$$

By $d_2 equivalence - negationslash 0, A_0 equivalence - negationslash 0$ and Lemma 2 . 2 we get from (2 . 6) that $\rho(h) = \rho(A_0) = \rho > 0$, then $h \not\equiv 0$.

Now suppose $d_2 \equiv 0, d_1 equivalence - negationslash 0$. Using a similar reasoning as above we get $h equivalence - negationslash 0$.

Finally , if $d_2 \equiv 0, d_1 \equiv 0, d_0 \not\equiv 0$, then we have $h = -d_0^2 \not\equiv 0$. □

3 . Proof of main results

Proof of Theorem 1 . 4 . Suppose that f is a solution of (1 . 2) with $\rho(f) = \infty$ and $\rho 2(f) = \rho$. Substituting $f'' = F - A_1 f' - A_0 f$ into g_f, we have

$$g_f - d_2 F = (d_1 - d_2 A_1) f' + (d_0 - d_2 A_0) f. \tag{3.1}$$

Differentiating both sides of (3 . 1) and using that $f'' = F - A_1 f' - A_0 f$, we obtain

$$g_f' - (d_2 F)' - (d_1 - d_2 A_1) F = [^{d_2 A_1^2 \; - \; (d_2 A 1)' \; - d_1 A 1}_{+ \; [d_2 A 0^{A 1} \; - (d_2 A 0\;)' \; -} - d_1 d_2 A_0^{A0}{}_{+} + d_{\prime 0}] d_0 f. + d_1'] f' \tag{3.2}$$

Then , by (1 . 3) , (1 . 4) , (3 . 1) and (3 . 2) , we have

$$\alpha_1 f' + \alpha_0 f = g_f - d_2 F, \tag{3.3}$$

$$\beta 1 f' + \beta 0 f = g_f' - (d_2 F)' - (d_1 - d_2 A_1) F. \tag{3.4}$$

Set

$$\begin{aligned} h = \alpha_1 \beta 0 - \alpha_0 \beta 1 \\ = (d_1 - d_2 A_1)(d_2 A_1^2 - (d_2 A_1)' - d_1 A_1 - d_2 A_0 + d_0 + d_1') \\ -(d_0 - d_2 A_0)(d_2 A_0 A_1 - (d_2 A_0)' - d_1 A_0 + d_0'). \end{aligned} \tag{3.5}$$

By the condition $h \not\equiv 0$ and (3 . 3) - (3 . 5) , we obtain

$$f = \frac{\alpha_1 (g_f' - (d_2 F)' - \alpha_1 F) - \beta 1 (g_f - d_2 F)}{h}. \tag{3.6}$$

If $\rho(g_f) < \infty$, then by (3 . 6) we obtain $\rho(f) < \infty$ and this is a contradiction . Hence

$$\rho(g_f) = \infty.$$

Now , we prove that $\rho 2(g_f) = \rho 2(f) = \rho$. By $g_f = d_2 f'' + d_1 f' + d_0 f$, we obtain $\rho 2(g_f) \leqslant \rho 2(f)$ and by (3 . 6) , we have $\rho 2(f) \leqslant \rho 2(g_f)$. Hence $\rho 2(g_f) = \rho 2(f) =$

ρ. □

Proof of Theorem 1 . 5 . Suppose that f is a solution of (1 . 2) with $\rho(f) = \infty$ and $\rho 2(f) = \rho$. Set $w(z) = g_f - \varphi$. Since $\rho(\varphi) < \infty$, then by Theorem 1 . 4 , we have $\rho(w) = \rho(g_f) = \rho(f) = \infty$ and $\rho 2(w) = \rho 2(g_f) = \rho 2(f) = \rho$. To prove $\text{-}\lambda_{(g_f} - \varphi) = \lambda(g_f - \varphi) = \infty$ and $\text{-}\lambda_{2(g_f} - \varphi) = \lambda_2(g_f - \varphi) = \rho$, we need to prove only $\text{-}\lambda_{(w)} = \lambda(w) = \infty$ and $\text{-}\lambda_{2(w)} = \lambda_2(w) = \rho$. By $g_f = w + \varphi$, and using (3 . 6) , we have

$$f = \frac{\alpha_1 w' - \beta 1^w}{h} + \psi(z), \tag{3.7}$$

 where $\alpha_1, \beta 1, h, \psi(z)$ are defined in (1 . 3) - (1 . 6) . Substituting (3 . 7) into equation (1 . 2) , we obtain

$$\frac{\alpha_1}{h} w''' + \phi 2^{w''} + \phi 1^{w'} + \phi 0^{w} = F - (\psi'' + A_1(z)\psi' + A_0(z)\psi) = A, \qquad (3.8)$$

where $\phi_j (j = 0, 1, 2)$ are meromorphic functions in D with $\rho(\phi_j) < \infty (j = 0, 1, 2)$. Since $\psi(z)$ is not a solution of (1 . 2) , it follows that $A \not\equiv 0$. Then , by Lemma 2 . 1 , we obtain $\bar{\lambda}_{(w)} = \lambda(w) = \rho(w) = \infty$ and $\bar{\lambda}_{2(w)} = \lambda_2(w) = \rho 2(w) = \rho$; i . e . ,

$$\bar{\lambda}_{(g_f} - \varphi) = \lambda(g_f - \varphi) = \infty \text{and} \bar{\lambda}_{2(g_f} - \varphi) = \lambda_2(g_f - \varphi) = \rho. \quad \square$$

Proof of Theorem 1 . 7 . By the hypotheses of Theorem 1 . 7 , all solutions of (1 . 2) are of infinite order . From (1 . 6) , we see that $\psi(z)$ is of finite order , then $\psi(z)$ is not a solution of equation (1 . 2) . By Theorem 1 . 5 , we obtain Theorem 1.7. □

Proof of Corollary 1 . 9 . By Theorem 1 . 2 , all solutions $f \not\equiv 0$ of (1 . 1 0) are of infinite order and satisfy

$$\rho(A_0) \leqslant \rho 2(f) \leqslant \max\{\rho M(A_0), \rho M(A_1)\}.$$

Also , by Lemma 2 . 3 , we have $hequivalence - negationslash$ 0. Then , by using Theorem 1 . 7 we obtain Corollary 1.9. □

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