Chapter 15
A Mathematical Model to Optimize Transport Cost and Inventory Level in a Single Level Logistic Network

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Abstract This paper proposes a mathematical model that minimizes transportation costs and optimizes distribution organization in a single level logistic network. The objective is to allocate customers to distribution centers and vehicles to travels in order to cut down the traveled distances, while observing the storage capacities of vehicles and distribution centers and covering the customers' needs. We propose a mixed integer programming formula that can be solved using Lingo 14.0. A digital example will be given in the end to illustrate the practicability of the model.

Keywords Distribution organization • Mixed integer programming • Single level logistic network • Transportation costs

15.1 Introduction

Supply chain is the succession of processes transforming raw materials into finished products delivered to the final customers, it consists of activities of supply, manufacturing, storage, distribution and sale. Transport is one of the main functions of supply chain which is present in several levels; to connect suppliers to factories, factories to distribution centers and the latter to customers. Connecting these various points via means of transport is what we call a logistic distribution network (see Fig. 15.1).

A logistic network consists of one or several levels; a single level logistic network is a network where there is a single intermediary between factories and customers, for example: distribution centers, whereas a multi level logistic network consists of several intermediaries between factories and customers, for example: distribution centers, centers of transfer, hubs, etc.

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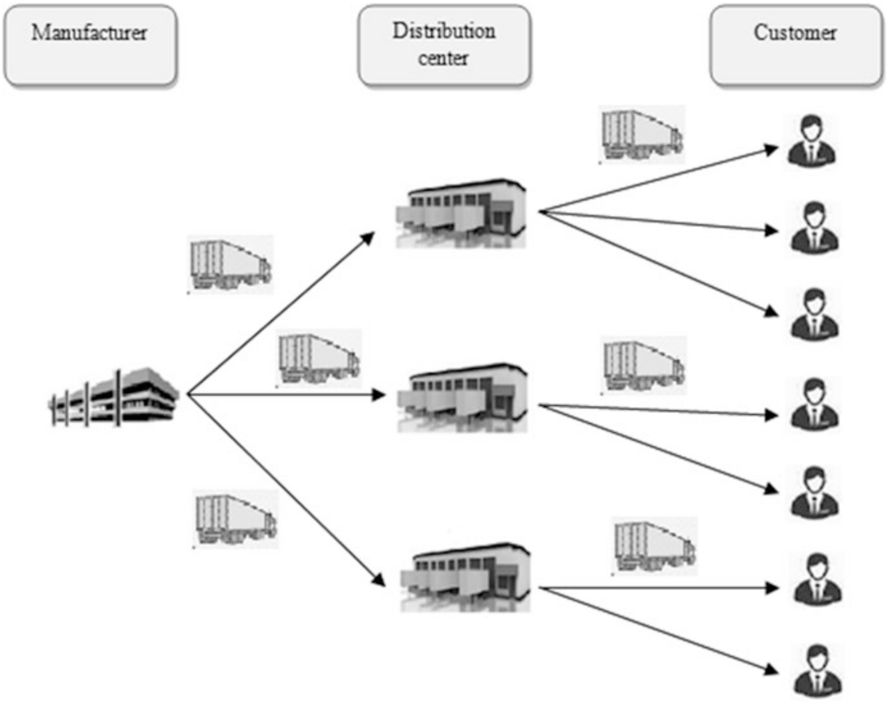

Fig. 15.1 Transport role in distribution network

Companies that manage distribution of their goods seek to cut down transport costs and avoid stock shortages at their distribution centers. To ensure a certain quality of customer service, companies have to manage the allocation of customers to distribution centers in a way that minimizes the costs of transport and considers the storage capacity of the vehicles as well as of the various distribution network's nodes.

The problem related to transport and distribution of goods ranges from vehicle routing problems such as the TSP (Travelling Salesman Problem) formulated by the mathematicians WR Hamilton and Thomas Kirkman in 1800 [1, 2], to problems that consider the interactions between the different activities like production, warehousing and transport to minimize the total costs [3] and problems of constructing the whole networks and thus to factories setting up and allocation of various nodes to customers [4, 5]. Vehicle Routing Problem has been an active area of research; Traveling Salesman Problem (TSP) focuses on finding the optimal route to visit a given number of cities while minimizing transportation cost [6, 7], the Vehicle routing problem (VRP), which is an extension of the TSP, was formulated in 1959 by Dantzig and Ramser [8], according to Laporte [9], this problem aims at building the optimal tours of pickup or delivery, from one or several warehouses

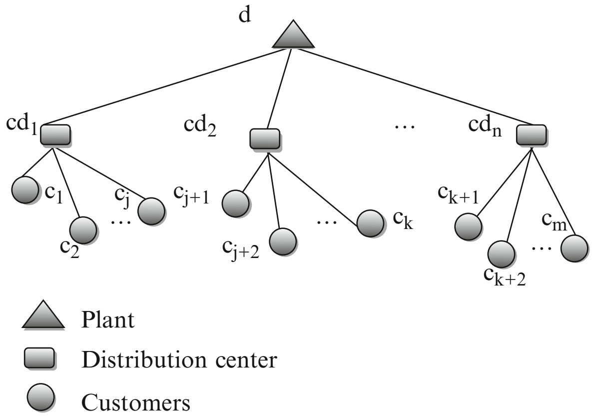

Fig. 15.2 A single level logistic network

towards a number of customers or cities that are geographically scattered, while respecting certain constraints. There exist four variants of the VRP [10]: VRP with Time Windows (VRPTW) [11], VRP with Pickup and Delivery (VRPPD) [12], the capacitated VRP (CVRP) [13] and the VRP with Backhauls (VRPB) [14].

The first algorithm to solve the VRP problem, was proposed by Clarke and Wright in 1964 [15], and since then, several methods were proposed and which are either exact methods that allow to find an optimal solution, or approximate methods that allow to obtain a solution to the problem but which is not optimal [16].

Since its introduction, the formulation of several models aiming at the optimization of the transport costs has been based on the VRP. Likewise, the proposed mathematical model is based on the VPR and addresses the minimization of transport costs as well as those of storage, both being parts of logistic distribution.

In the following section, we will begin by presenting our mathematical model, then, we will apply it to a real case and solve it by the Lingo 14.0 software to test its reliability.

We consider a logistic network consisted of one plant, n distribution centers and m customers (see Fig. 15.2). The first objective is to allocate customers to distribution centers and vehicles to travels so as to minimize the distances to travel, and ultimately the transport costs. The second objective is to minimize the storage costs at the distribution centers by minimizing the stored quantities while respecting the daily quantities that can be delivered to every center and satisfy the customers' needs (see Fig. 15.3).

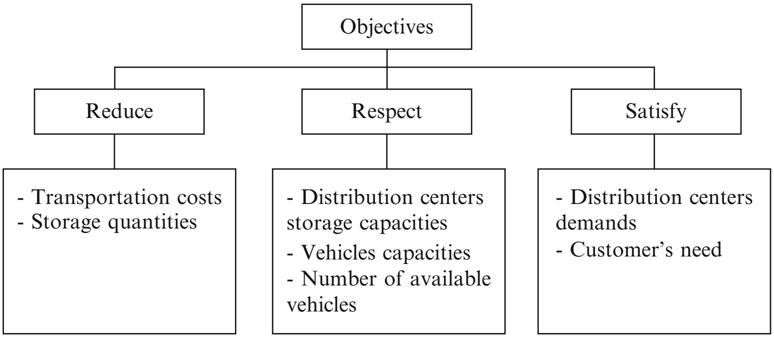

Fig. 15.3 Objectives of the model

15.2 Mathematical Formulation

Sets

I: Collection of distribution centers $i \in I, i = 1, 2, \ldots, n$;
M: Set of customers $j, \quad j \in M, j = 1, 2 \ldots m$;
T: Set of periods $t, \quad t \in T, t = 1, 2 \ldots t'$;
C_h: Set of vehicles vh with a capacity h;
C: Set of C_h;
Parameters

d_{ip}: Distance between plant p and center i
d_{ij}: Distance between center i and customer j
c_{vh}: Transportation cost per km for a vehicle vh
c_{si}: Unit cost of storage per day at distribution center i
n_h: Number of vehicles of category C_h
cap_{vh}: Vehicle vh capacity
b_i^t: Center i demand on day t
c_i: Storage capacity at center i
bes_j^t: Customer j demand on day t
stk_i^t: Available stock at center i on day t

Decision Variables

x_{ip}^t: Quantity to deliver from the plant p to center i on day t
y_{ij}^t: Quantity to deliver from center i to customer j on day t

$$l_{vh}^{it} = \begin{cases} 1 \text{ if vehicle vh visits center i on day t} \\ 0 \text{ else} \end{cases}$$

$$l_{vh}^{ijt} = \begin{cases} 1 \text{ if vehicle vh travels from center i to customer j on day t} \\ 0 \text{ else} \end{cases}$$

We assume all parameters are nonnegative.
Objective function:

$$\begin{aligned} Min \sum\nolimits_{v \in Ch} \sum_{t=1}^{t'} \sum_{i \in I} l_{vh}^{it} x_{ip}^{t} d_{ip} c_{vh} + \sum\nolimits_{v \in Ch} \sum_{t=1}^{t'} \sum_{i \in I} \sum\nolimits_{j \in M} l_{vh}^{ijt} y_{ip}^{t} d_{ij} c_{vh} \\ + \sum_{t=1}^{t'} \sum_{i \in I} c_{si} \left(x_{ip}^{t} - b_{i}^{t} + stk_{i}^{t} \right). \end{aligned} \tag{15.1}$$

Subject to

$$x_{ip}^{t} - b_{i}^{t} \leq c_i \quad \forall i \in I \quad \forall t \in T. \tag{15.2}$$

$$b_{i}^{t} \geq x_{ip}^{t} + stk_{i}^{t} \quad \forall i \in I \quad \forall t \in T. \tag{15.3}$$

$$stk_{i}^{t} = stk_{i}^{t-1} + x_{ip}^{t} - b_{i}^{t} \quad \forall i \in I \quad \forall t \in T. \tag{15.4}$$

$$\sum\nolimits_{i \in I} y_{ij}^{t} = bes_{j}^{t} \quad \forall i \in I \quad \forall j \in M \quad \forall t \in T. \tag{15.5}$$

$$\sum\nolimits_{vh \in C_h} l_{vh}^{it} \leq n_h \quad \forall i \in I \quad \forall t \in T \quad \forall vh \in C_h. \tag{15.6}$$

$$l_{vh}^{it} x_{ip}^{t} \leq cap_{vh} \qquad \forall i \in I \qquad \forall t \in T \qquad \forall vh \in C_h. \tag{15.7}$$

$$l_{vh}^{ijt} y_{ij}^{t} \leq cap_{vh} \quad \forall i \in I \quad \forall j \in M \quad \forall t \in T \quad \forall vh \in C_h. \tag{15.8}$$

$$l_{vh}^{it}, l_{vh}^{ijt} \in \{0, 1\} \quad \forall i \in I \quad \forall j \in M \quad \forall t \in T \quad \forall vh \in C_h. \tag{15.9}$$

The objective function (15.1) expresses the cost to be minimized and which is the sum of:

- Travelling costs from the plant to distribution centers;
- Travelling costs from centers to the customers;
- Storage costs at the distribution centers.

Constraint (15.2) assures the respect of the storage capacity of every distribution center.

Constraint (15.3) assures that the daily need for every distribution center is satisfied.

Constraint (15.4) calculates the quantity available in every distribution center.

Constraint (15.5) assures that the daily need of every customer is satisfied.

Constraint (15.6) assures the respect of the number of vehicles available in each category.

Constraints (15.7) and (15.8) assure the respect of each vehicle capacity.

15.3 Illustrative Example

To illustrate our model, we apply it to a network consisted of a single plant, 4 distribution centers and 13 customers. Table 15.1 includes storage parameters. There are two categories of vehicles for travels linking plant to centers and two other categories for travels linking centers to customers. Table 15.2 represents the characteristics of different vehicles, we note that when sending a vehicle to a center, it is completely filled, even if the sent quantity exceeds the center need, what explains the existence of stock.

Tables 15.3 and 15.4 represent respectively distances between plant and various distribution centers, and distances between the latter and customers. Tables 15.5 and 15.6 represent respectively the daily needs of distribution centers and of customers over a period of 4 days.

Table 15.1 Parameters values

Parameter	Value
c_{si}	0.20
c_i	500

Table 15.2 Characteristics of each type of vehicle vh

vh	A	B	C	D
c_{vh}	0.21	0.21	0.20	0.20
n_h	2	4	8	10
cap_{vh}	1,000	600	350	200

Table 15.3 Distances between the plant and distribution centers

	cd_2	cd_3	cd_4	cd_1
Plant	511	0	291	369

Table 15.4 Distances between distribution centers and customers

	cd_1	cd_2	cd_3	cd_4
c_1	419	99	390	469
c_2	172	351	642	721
c_3	651	133	166	237
c_4	719	259	204	93
c_5	303	241	485	611
c_6	746	23	60	267
c_7	614	93	198	281
c_8	772	314	29	422
c_9	439	72	361	433
c_{10}	735	217	82	221
c_{11}	87	483	773	818
c_{12}	907	411	120	423
c_{13}	910	390	286	60

Table 15.5 Daily distribution centers' need during period T

	j_1	j_2	j_3	j_4
cd_1	822	832	840	838
cd_2	793	1,007	985	1,015
cd_3	508	531	516	513
cd_4	476	472	460	481

Table 15.6 Daily customers' need during period T

	j_1	j_2	j_3	j_4
c_1	300	311	298	288
c_2	302	298	288	278
c_3	200	188	185	186
c_4	150	147	156	148
c_5	340	345	329	330
c_6	188	201	210	198
c_7	347	300	321	311
c_8	150	165	140	160
c_9	250	248	211	200
c_{10}	139	128	148	144
c_{11}	180	189	223	230
c_{12}	170	165	166	155
c_{13}	187	197	156	189

Table 15.7 Optimal quantities to be sent to the distribution centers during period T

	j_1	j_2	j_3	j_4
cd_1	1, 000	1, 000	1, 000	1, 000
cd_2	1, 000	1, 000	1, 000	1, 000
cd_3	600	600	600	600
cd_4	600	600	600	600

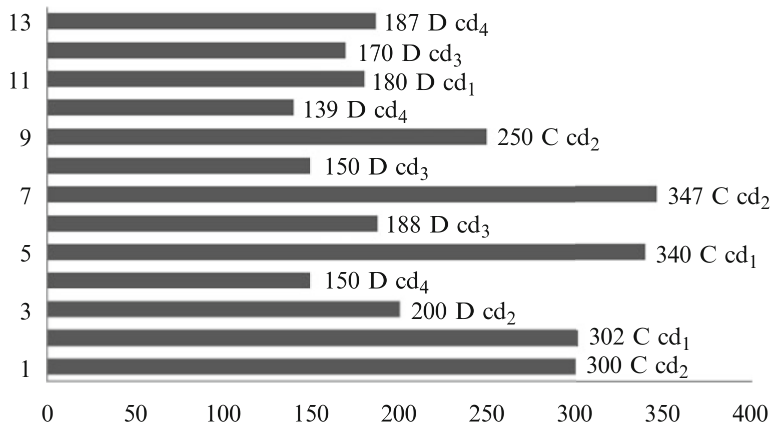

Fig. 15.4 Affectations on day J_1

15.3.1 Discussion

We solve this problem using a Mixed Integer Linear Programming solver LINGO 14.0 [17] on an Acer Aspire ONE D255 1.00 GHz machine, running Windows 7 Starter Edition. Results are obtained in 0.56 s, and the objective value is 901032.1.

Table 15.7 represents the optimal quantities to be sent to distribution centers during period T and which meet their needs. Figures 15.4, 15.5, 15.6, and 15.7 represent each the affectation of customers to distribution centers, optimal quantities to be sent on every day of period T and which category of vehicle to use.

We notice that obtained results respect the various constraints of our example, which are the storage capacity of distribution centers and the needs of the final customers. According to these results, we can easily deduct the optimal affectation of customers to distribution centers, which is, in this example represented in Fig. 15.8.

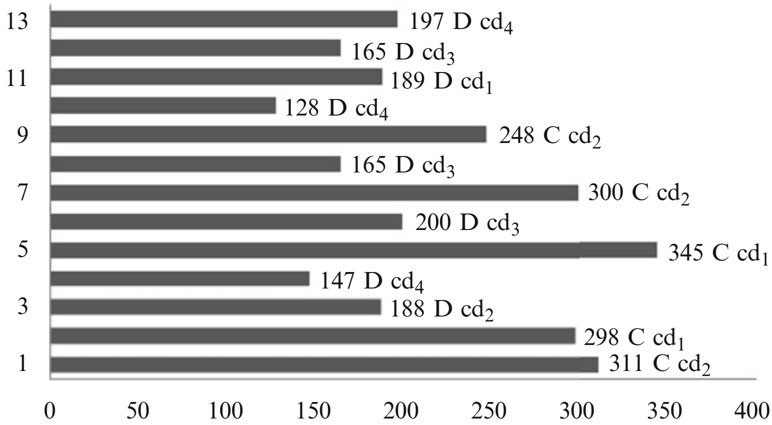

Fig. 15.5 Affectations on day J_2

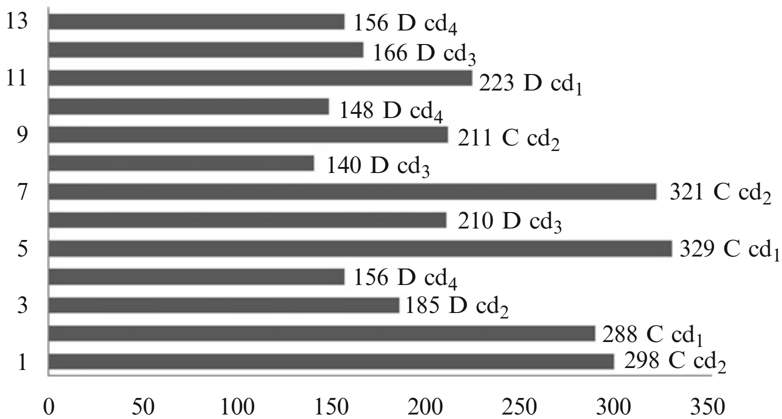

Fig. 15.6 Affectations on day J_3

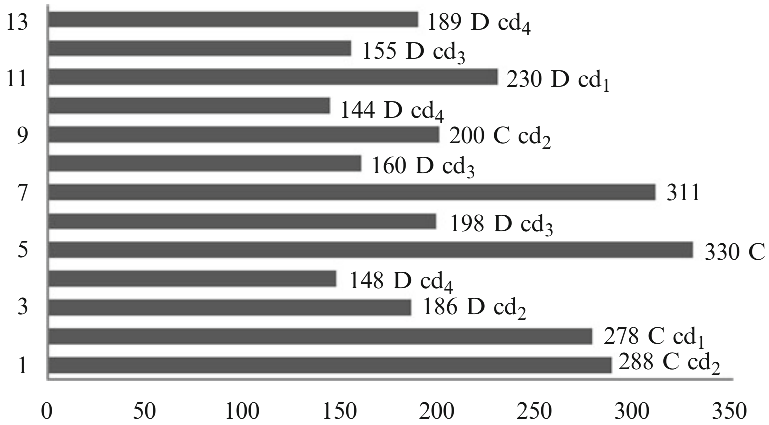

Fig. 15.7 Affectations on day J_4

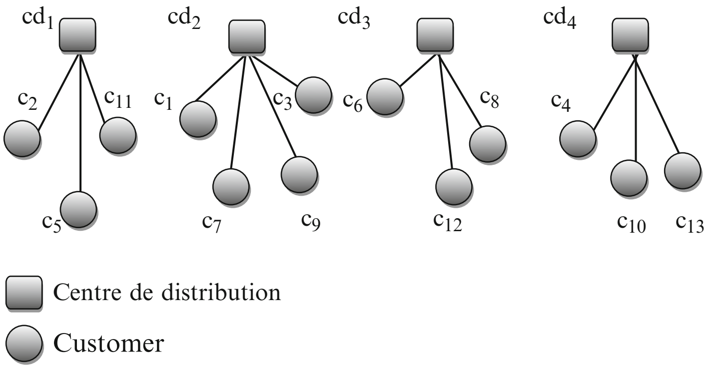

Fig. 15.8 Affectation of customers to distribution centers

15.4 Conclusion and Perspectives

The number of scientific publications handling transport problems continues to increase, so proving the importance of this function of supply chain. In this paper, we investigate the optimization of the distribution problem, the objective is to minimize both the traveled distances and the storage level, and allocate vehicles to travels. We relied on the vehicle routing problem VRP to develop our mathematical formula.

In this work, a single level logistic network is considered to apply our model. As perspective, we can consider a multi level logistic network, the model can be easily developed and applied in that case.

References

1. Davendra, D.: Traveling Salesman Problem, Theory and Applications. InTech, Hyderabad (2010)
2. Exnar, F., Machac, O.: The travelling salesman problem and its application in logistic practice. WSEAS Trans. Bus. Econ. **8**(4), 163–173 (2011)
3. Fahimnia, B., Luong, L., Marian, R.: Optimisation/simulation modeling of the integrated production-distribution plan: an innovative survey. WSEAS Trans. Bus. Econ. **5**(3), 44–57 (2008)
4. Chopra, S.: Designing the distribution network in a supply chain. Transp. Res. Part E Logist. Transp. Rev. **39**, 123–140 (2003)
5. Guerra, L., Murino, E., Romano, E.: The location-routing problem: an innovative approach. In: 6th WSEAS Transactions on System Science and Simulation in Engineering, Venice, Italy, 21–23 November 2007
6. Laporte, G.: The traveling salesman problem: an overview of exact and approximate algorithms. Eur. J. Oper. Res. **59**, 231–247 (1992)
7. Dantzig, G.B., Fulkerson, D.R., Johnson, S.M.: The solution of a large-scale traveling salesman problem. Oper. Res. **2**, 393–410 (1954)

8. Dantzig, G.B., Ramser, J.H.: The truck dispatching problem. Manag. Sci. **6**, 80–91 (1959)
9. Laporte, G.: The vehicle routing problem: an overview of exact and approximate algorithms. Eur. J. Oper. Res. **59**, 345–358 (1992)
10. Toth, P., Vigo, D.: The Vehicle Routing Problem. SIAM Monographs on Discrete Mathematics and Applications. Society for Industrial and Applied Mathematics, Philadelphia (2001)
11. Solomon, M.: Algorithms for the vehicle routing problem with time windows. Transp. Sci. **29**(2), 156–166 (1995)
12. Parragh, S.N., Doerner, K.F., Hartl, R.F.: A survey on pickup and delivery problems, part I: transportation between customers and depot. J. Betriebswirt. **58**, 21–51 (2008)
13. Ralphs, T.K., Kopman, L., Pulleyblank, W.R., Trotter Jr., L.E.: On the capacitated vehicle routing problem. Math. Program. **94**(2–3), 343–359 (2003)
14. Toth, P., Vigo, D.: An exact algorithm for the vehicle routing problem with backhauls. Transp. Sci. **31**(4), 372–385 (1997)
15. Clarke, G., Wright, J.W.: Scheduling of vehicles from a central depot to a number of delivery points. Oper. Res. **12**, 568–581 (1964)
16. Laporte, G.: What you should know about the vehicle routing problem. Nav. Res. Logist. **54**(8), 811–819 (2007)
17. LINGO: The Modeling Language and Optimizer. LINDO Systems Inc., Chicago (2013)