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Statistical estimation of GNSS pseudo-range errors

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Abstract

Technology of satellite has many advantages over long instruments used nowadays and gives unparalleled precision compared to other systems. Nonetheless, it is full of problems related to the spread in the atmosphere, barriers in the receiving medium, the instability of the clocks used or the receiver's electronic noise of the errors that these phenomena cause may lead to inaccuracies of over to tens of meters. This paper describes methods for estimating the pseudo-range errors based on different statistical filtering. The Rao-Blackwellized filter has given interesting results comparing to the extended kalman filter, but the particle filter with kalamn filter proposal is much better than many other particle filtering algorithms.

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1. Introduction

The accuracy of GNSS (Global Navigation Satellite Systems) positioning is a function of the geometry of the satellite's distribution in the data collection and the accuracy of measurements, this latter depends on the type of observations and the resolution of the measurements made by the receiver. Moreover, errors inherent in GNSS system also influence the positioning quality[1]. GNSS performance depends greatly on the signal propagation environment. The propagation characteristics in the atmosphere are fairly well known. By cons, it is more difficult to predict and analyze the impact of the environment close to the antenna [2].This paper deals with the modelling of

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positioning errors to improve location accuracy. For the continuation of this work different statistical methods envisaged for a GNSS positioning.

Many complex fusion methods are used to improve the precision of the pseudo-range estimation. These methods include continuously the available measurements in some optimal sense. Statistical adjustment by LS (Least Square) does not take account of any information on the measures and noise to correct its estimate position[3]. Algorithms modified on the basis of LS have been proposed -such as the ILS (Iterative Least Square) algorithm-[4] to avoid searching the approximate initial position and achieve higher accuracy than that of LS algorithm. However, the accuracy is much lower than that of the EKF (Extended Kalman Filter) algorithm [5, 6]. The KF is the most popular, it uses the a priori information and the kinematics equations to compute an optimal position which minimizes the MSE, but in our case, the classical KF(Kalman Filter) cannot be used because the expression of the pseudo-range is weakly nonlinear. Then, we must use the non-linear version of this filter. For this filter, the state noise and measurement noise are independent, white and Gaussian. The performance of this filter is, however, strongly degraded when assumptions about the noises are not respected or if the non-linearity of the models is strong during dynamic changes or disturbance measurements.

Unlike previous filters, the PF (Particular Filter) is not the assumption of white Gaussian noise and linear systems. It allows solving problems of multi-modal noise[7]. The principle of this method is to generate a large number of samples from a probability distribution. For each sample, a weight is associated, which provides a better estimation of the state. As EKF, the PF is also part of the family of suboptimal filters. This type of filter makes an estimation of the unknown state based on SMC (Sequential Monte Carlo), itself based on a representation by mass point's probability densities[8]. More complete descriptions of this method have been made in [8] and [9]. One of the big disadvantages of this filter is the large computational cost. In this paper, we introduce the main statistical filtering concepts necessary to understand the work presented. We presented the modeling state and the main estimators that can be used for GNSS positioning.

This paper is organized as follows. First, we briefly review how to determine the position of a GPS Global Positioning System) receiver. Then, we explain the different statistical method. Next, how to evaluate the quality of the observations and techniques of weighting them according to their quality are discussed. Finally, the simulation results and comparison with other methods are presented.

2. Problem formulation

Measuring pseudo range is an estimate of the receiver-satellite distance derived from the measurement of the propagation time of the signal between the satellite and the receiver. This time is measured by the cross-correlation between the replica of the PRN(Pseudo-random Noise) code, C/A(Clear /Acquisition code) or P(Precision code), generated by the receiver and the PRN code transmitted by the satellites[10]. In actual propagation conditions, a number of phenomena lead to errors in the pseudo range. Then, according to the literature and in Cartesian coordinates, the pseudo range measured on the signal from satellite i is as follows:

$$\rho_s = \sqrt{(x_s - x_r)^2 + (y_s - y_r)^2 + (z_s - z_r)^2} + c(\delta t_r - \delta t_s) + e_s \tag{1}$$

$(x_s; y_s; z_s)$ The coordinates of the satellite s and $(x_r; y_r; z_r)$ the coordinates of the receiver respectively at the time of transmission and reception of the signal, : the celerity, δt_r ,δt_s : the receiver and satellite clock bias respectively, e_s: The pseudo-range error for satellite s.

$$e_s = I_s + T_s + M_s + N \tag{2}$$

I_s And T_s are the ionospheric and tropospheric errors, M_s is the error caused by signal reflections (multipath errors), and N is the noise receiver.We set $\Delta t = (\delta t_r - \delta t_s)$ and considering $f(x_s, y_s, z_s)$ the nonlinear term of equation (1):

$$f(x_s, y_s, z_s) = \sqrt{(x_r - x_s)^2 + (y_r - y_s)^2 + (z_r - z_s)^2} \tag{3}$$

To linearize f, we will use the limited development in Taylor series, this development cannot be done only in the vicinity of a known position (origin or previous position). We need to redefine the position estimate based on the known position as follows:

$$\begin{cases} x_r = x_{t_0} + \Delta x_r \\ y_r = y_{t_0} + \Delta y_r \\ z_r = z_{t_0} + \Delta z_r \end{cases} \tag{4}$$

With $(x_{t_0}, y_{t_0}, z_{t_0})$ is the initial position and $(\Delta r, \Delta y_r, \Delta z_r)$ is the update of the position relative to the initial position at time t_0. Using Taylor's series and limiting to the first order the equation (2) and (3) can be elaborated as below:

$$f(x_r, y_r, z_r) = f(x_{t_0}, y_{t_0}, z_{t_0}) + \frac{\partial f(x_{t_0}, y_{t_0}, z_{t_0})}{\partial x_{t_0}} \Delta x_r + \frac{\partial f(x_{t_0}, y_{t_0}, z_{t_0})}{\partial y_{t_0}} \Delta y_r + \frac{\partial f(x_{t_0}, y_{t_0}, z_{t_0})}{\partial z_{t_0}} \Delta z_r \tag{5}$$

Developing the partial derivative of the equation (4) and posing $d_{s,0}$ as the approximation of the satellite-receiver distance:

$$\mathrm{d_{s,0}} = \sqrt{(\mathrm{x_s} - \mathrm{x_0})^2 + (\mathrm{y_s} - \mathrm{y_0})^2 + (\mathrm{z_s} - \mathrm{z_0})^2} \tag{6}$$

The expression of the pseudo range becomes:

$$\rho_s = d_{s,0} - \frac{x_s - x_0}{d_{s,0}} \Delta x_r - \frac{y_s - y_0}{d_{s,0}} \Delta y_r - \frac{z_s - z_0}{d_{s,0}} \Delta z_r + c\Delta t_r \tag{7}$$

To estimate a position, a single pseudo-range measurement is not sufficient. At least four different measurements to calculate a position in 3D. In practice, the receiver takes into account all the pseudo-measures available in order to compensate for any disruption of signals or any bad configuration. Therefore, it is necessary to reformulate the equation (7) in matrix form by taking into account all available pseudo-ranges. This reformulation is given as follows:

$$\begin{bmatrix} \rho_1 - d_{1,0} \\ \vdots \\ \rho_n - d_{n,0} \end{bmatrix} = \begin{bmatrix} -\frac{x_1 - x_0}{d_{1,0}} & -\frac{y_1 - y_0}{d_{1,0}} & -\frac{z_1 - z_0}{d_{1,0}} & c \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{x_n - x_0}{d_{n,0}} & -\frac{y_n - y_0}{d_{n,0}} & -\frac{z_n - z_0}{d_{n,0}} & c \end{bmatrix} \begin{bmatrix} \Delta x_r \\ \Delta y_r \\ \Delta z_r \\ \Delta t_r \end{bmatrix} \tag{8}$$

To simplify the notation we put $\mathbf{Y}_{\mathbf{s}_i} = \rho_{s_i} - d_{s,0_i}$, the matrix representation of (8) is identified as:

$$Y = HX \tag{9}$$

The equation (9) is the matrix form of the observation equation; we will need to express the evolution equation in this form. For this, firstly, it is necessary to express the evolution of the antenna's position.

For the model of state, we have to describe the dynamics of a system in motion. The equation of motion along the uniformly accelerated rectilinear axis x is given by:

$$\begin{cases} x_t = x_{t-1} + v_{t-1}\Delta t + \frac{1}{2} a \Delta t^2 \\ v_t = \dot{x}_t = v_{t-1} + a\Delta t \\ a_t = \ddot{x}_t = a = constant \end{cases} \tag{10}$$

Where Δt is the sampling step, a_t is the acceleration and v_t the speed. Many cases are possible for a rectilinear movement.I

$$\begin{pmatrix} x_t \\ y_t \\ z_t \\ c\Delta t_r \\ \dot{x}_t \\ \dot{y}_y \\ \dot{z}_t \\ c\dot{\Delta} t_r \\ \ddot{x}_t \\ \ddot{y}_t \\ \ddot{z}_t \\ c\ddot{\Delta} t_r \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & \Delta t & 0 & 0 & 0 & 1/2\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \Delta t & 0 & 0 & 0 & 1/2\Delta t^2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \Delta t & 0 & 0 & 0 & 1/2\Delta t^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \Delta t & 0 & 0 & 0 & 1/2\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ z_{t-1} \\ c\Delta t_r \\ \dot{x}_{t-1} \\ \dot{y}_{y-1} \\ \dot{z}_{t-1} \\ c\dot{\Delta} t_r \\ \ddot{x}_{t-1} \\ \ddot{y}_{t-1} \\ \ddot{z}_{t-1} \\ c\ddot{\Delta} t_r \end{pmatrix} \quad (11)$$

$$X_t = FX_{t-1} \quad (12)$$

2.1. Position estimation using ILS

In this part, we describe the navigation equation solution based on the ILS method. Let i be the number of the iterations, $(i = 1, 2, \cdots, imax)$. The increments $(\Delta x_{r_i}, \Delta y_{r_i}, \Delta z_{r_i})$ update the receiver coordinates as follows:

$$\begin{cases} x_{r_i} = x_{t_{0_i}} + \Delta x_{r_i} \\ y_{r_i} = y_{t_{0_i}} + \Delta y_{r_i} \\ z_{r_i} = z_{t_{0_i}} + \Delta z_{r_i} \end{cases} \quad (13)$$

Taking into account the errors of pseudo range, we put:

$$e_{s_i} = I_{s_i} + T_{s_i} + M_{s_i} + N_i \quad (14)$$

And the equation (7) becomes:

$$\begin{bmatrix} Y_{1_i} \\ \vdots \\ Y_{n_i} \end{bmatrix} = \begin{bmatrix} -\frac{x_1 - x_{0_i}}{d_{1,0_i}} & -\frac{y_1 - y_{0_i}}{d_{1,0_i}} & -\frac{z_1 - z_{0_i}}{d_{1,0_i}} & c \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{x_n - x_{0_i}}{d_{n,0_i}} & -\frac{y_n - y_{0_i}}{d_{n,0_i}} & -\frac{z_n - z_{0_i}}{d_{n,0_i}} & c \end{bmatrix} \begin{bmatrix} \Delta x_{r_i} \\ \Delta y_{r_i} \\ \Delta z_{r_i} \\ \Delta t_{r_i} \end{bmatrix} + \begin{bmatrix} e_{1_i} \\ \vdots \\ e_{n_i} \end{bmatrix} \quad (15)$$

$$Y_i = H_i X_i + e_{s_i} \quad (16)$$

Then we obtain the ILS problem:

$$min_{X_i} \|Y_i - H_i X_i\| = e_{s_i} \quad (17)$$

To solve the equation, we need to find $\mathbf{X}_i$ which minimizes the length of the error $\mathbf{V}_i$ as follows:

$$\|V_i\|^2 = (Y_i - H_i X_i)^T (Y_i - H_i X_i) \quad (18)$$

2.2. The EKF approach

The EKF uses the non-linear system for computing the predicted state estimate $\widehat{\mathbf{X}}(t+1, t)$ and the non-linear measurement model for the predicted measurement $\widehat{\mathbf{Y}}(t+1, t)$ [8]. Then the state model takes on the KF can be generally expressed as follows:

$$\begin{cases} Y_t = H_t X_t + e_{s_t} + V_t \\ X_{t+1} = F X_t + e_{s_t} + W_t \end{cases} \quad (19)$$

Where **F** is a known matrix with proper dimensions, $\mathbf{W_t}$ and $\mathbf{V_t}$ are uncorrelated, zero-mean, white random processes, **Q** is a covariance matrices and , such that:

$$E\{W_t\} = 0, E\{V_t\} = 0 \quad (20)$$

$$E\left\{\begin{pmatrix} W_t \\ V_t \end{pmatrix} (W_s^T | V_s^T)\right\} = \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix} \delta_{ts} \quad (21)$$

$$E\{W_t X_s^T\} = 0,\ E\{V_t X_s^T\} = 0,\ s \leq t \quad (22)$$

$$X_0 \sim N(x_0, \in_0) \quad (23)$$

The algorithm of the EKF is given as follows:
Predicted and filtered estimations:

$$\hat{X}_{t+1|t} = F\hat{X}_{t|t} \quad (24)$$

$$\hat{X}_{t|t} = \hat{X}_{t|t-1} + K_t\left[Y_t - H\hat{X}_{t|t-1}\right] \quad (25)$$

Kalman Gain K:

$$K_t = P_{t|t-1}H^T\left[HP_{t|t-1}H^T + R\right]^{-1} \quad (26)$$

Predicted **P** and filtered error covariance matrices:

$$P_{t+1|t} = FP_{t|t}F^T + Q \quad (27)$$

$$P_{t|t} = P_{t|t-1} - K_t H P_{t|t-1} \quad (28)$$

Initial state:

$$\hat{X}_{0|-1} = X_0,\ P_{0|-1} = \in_0 \quad (29)$$

2.3. The Rao-blackwellization filter

Particle filtering methods construct a point mass representation of a distribution from a set of random samples (called particles) that explore the state-space [11].

The state vector $X_t = \left[x_{1,t}, x_{2,t}\right]^T$ with $x_{2,t}$ a part of $\mathbf{X_t}$ which is conditionally linear and Gaussian on $x_{1,t}$. The main idea of RBPF (Rao-Blackwellization Particular Filter) is to exploit this decomposition of the state vector [12]. In this case, the first part of the filter consists in providing a PF on $x_{1,t}$ then updating $x_{2,t}$ using $p\left(x_{2,t}\middle|x_{1,t}, Y_t\right)$. As $x_{2,t}$ follows conditionally a linear Gaussian model on $x_{1,t}$, $p\left(x_{2,t}\middle|x_{1,t}, Y_t\right)$ is Gaussian and KF can be used to update $x_{2,t}$. According to the cases, KF may be replaced by an EKF or UKF(Unscented Kalman Filter). The advantage of this method is that it reduces the error variance. Since $x_{2,t}$ is associated with each particle$\tilde{x}_{1,t}$, it is necessary to make a set of N steps of Kalman to update $x_{2,t}$. The computational cost is more important for this method than the previous filter versions. The Algorithm is as follows:

Generating the first part of the state vector and considering the importance density q and N samples x^i

$$\tilde{x}_{1,t} \sim q\left(x_{1,t} \middle| x^i_{1,t:t-1}\right) \tag{30}$$

Generating the second part of the estimated state vector conditionally to the first by a Kalman step

$$\left[\hat{x}^i_{2,t}\left(\tilde{x}^i_{1,t}, x^i_{1,1:t-1}\right), \hat{P}^i_{2,t}\left(\tilde{x}^i_{1,t}, x^i_{1,1:t-1}\right)\right] \tag{31}$$

Calculating the weight assigned to the particle

$$\widetilde{w}^i_t = \frac{p\left(y_t \middle| \tilde{x}_{1,t}, x^i_{1,1:t-1}, y_{1:t-1}\right).\, p\left(\tilde{x}_{1,t} \middle| x^i_{1,1:t-1}\right)}{q\left(\tilde{x}_{1,t} \middle| x^i_{1,1:t-1}\right)} \tag{32}$$

Calculating the sum of the weights

$$W = \sum_{i=1}^{N} \widetilde{w}^i_t \tag{33}$$

Performing the resampling

$$\hat{x}_{2,t} = \sum\nolimits_{i=1}^{N} w^i_t . x^i_{2,t}(\tilde{x}_{1,t}, x^i_{1,1:t-1}) \tag{34}$$

3. SIMULATION RESULTS

3.1. EKF vs. ILS performance

We used the EKF function with the application of GPS navigation. Results presented in the paper are of long-term static data from several stations around the world with different GPS receivers, as well as from kinematic experiments. The pseudo range and satellite position of a GPS receiver at fixed location for a period of 25 seconds is provided. Respecting the result of each algorithm, the position error of x, y and z are respectively shown in Table 2 and Table 3.

Table.1. Position error of each axis for different algorithms (m).

Axis	Algorithm	Mean Value	Standard deviation
	ILS	-0.9041	7.568
x	EKF	-0.0069	1.555
	ILS	-1.5654	4.437
y	EKF	-0.00841	1.827
	ILS	4.2109	5.394
z	EKF	-0.0013	2.388

Combining the statistic data in Table 1 with the figure 1, the position error of ILS algorithm are within 12 m; the position error of EKF algorithm are within 3 m. Compared with ILS algorithm, the precision of position EKF algorithm are much higher.

3.1. RBPF performance

Following the simulation of positioning errors of an aircraft, the pseudo-ranges corresponding to the satellites visible from the aircraft are evaluated. Beforehand, the positions of the satellite in their orbits have been calculated all along the trajectory. Figure 2 shows different states of a particle filter which generates a result of estimation; we can see that RBPF coincide better with the true state. Another advantage of RBPF classification algorithms is that they give entire probability estimates of class belonging as shown in Figure 3.

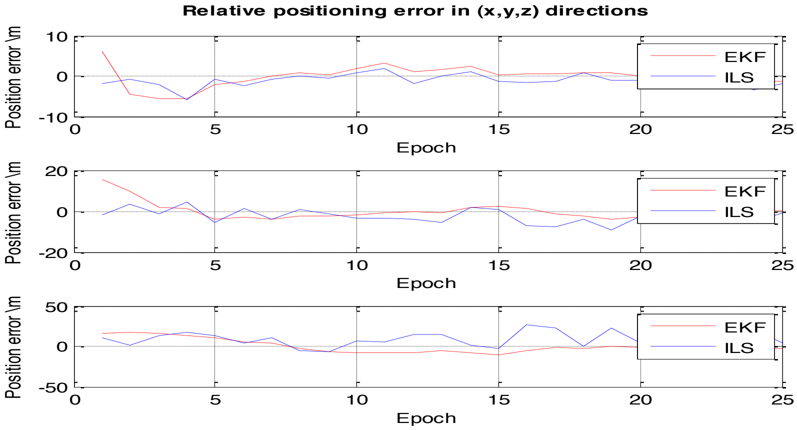

Fig. 1. Comparison of tracking performances using EKF and ILS algorithms.

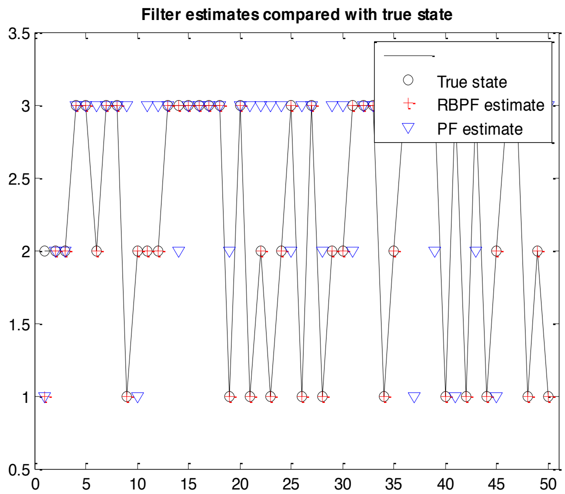

Fig. 2. True vs. predicted state by PF and RBPF.

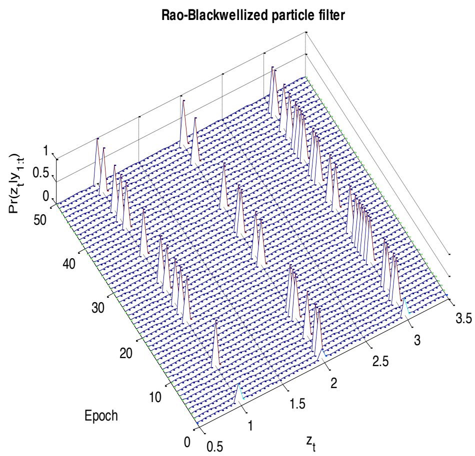

Fig3.Predective density

3.2. *RBPF vs. EKF performance*

According to the figure 5, the RBPF clearly outperforms the EKF. Specifically, the EKF suffers from an important track loss when an abrupt change occurs in the corresponding parameter. Table 2 shows the averaged MSE (Mean Square Error) values corresponding to the tracking shown in Fig. 1 averaged over 100 repeated Monte-Carlo runs. It confirms the finding in Fig. 1 that the EKF diverges quickly while the RBPF does not.

Table.2. the mean square error and variance after nonlinear filtering algorithm.

Algorithm	Mean square error	
	Mean	Variance
EKF	0.36969	0.0161216
PF	0.19089	0.041927
RBPF	0.015654	0.0004159

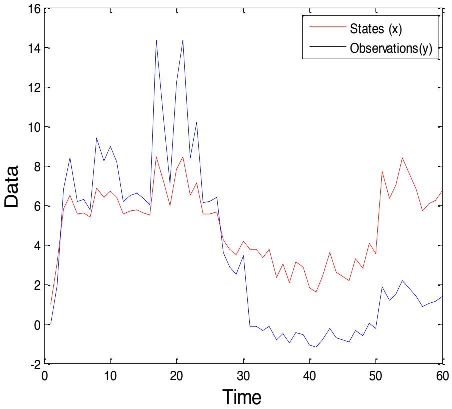

Fig. 4. the simulation of state and observation.

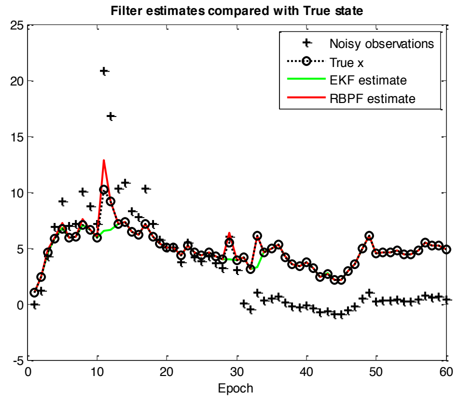

Fig.5. The position error for EKF and RBPF

4. Conclusion

Currently, the GPS is the only operational satellite positioning system; the algorithms are mainly tested on simulated GPS data. An opening on future GNSS systems will however made by testing the algorithms with data Simulated GPS-Galileo. RBPF attain good results in solving the nonlinear and non-Gaussian problems. Experimental results reveal that the RBPF algorithm is much better than many other particle filtering algorithms. RBPF algorithm provides a new method for solving the problem in the nonlinear filtering field. In the next step, we will solve the problem with mixed Kalman particle filter (MKPF) target with comprehensive algorithms and parameters estimation problem of nonlinear systems.

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