Electronic Journal of Differential Equations , Vol . 2004 (2004) , No . 1 38 , pp . 1 – 8 .
ISSN : 1 72 - 6691 . URL : http : / / ejde . math . txstate . edu or http : / / ej de . math . unt . edu
ftp ejde . math . txstate . edu (login : ftp)

EXISTENCE RESULTS FOR ELLIPTIC SYSTEMS INVOLVING CRITICAL SOBOLEV EXPONENTS

MOHAMMED BOUCHEKIF , YASMINA NASRI

ABSTRACT . In this paper , we study the existence and nonexistence of positive solutions of an elliptic system involving critical Sobolev exponent perturbed by a weakly coupled term .

1 . INTRODUCTION

We establish conditions for existence and nonexistence of nontrivial solutions to the system

$$\begin{aligned} -\Delta u &= (\alpha+1)u^{\alpha}v^{\beta+1} + \mu(\alpha'+1)u^{\alpha'}v^{\beta'+1} \quad \text{in}\Omega \\ -\Delta v &= (\beta+1)_u u\alpha^{+1}_{>0,}{}^{v^{\beta}} +_v \mu^{(\beta\prime}_{>0} + 1\text{in}^{)u^{\alpha'+1}v^{\beta'}}_{\Omega} \quad \text{in}\Omega \\ u &= v = 0 \quad \text{on}\partial\Omega, \end{aligned} \tag{1.1}$$

where Ω is a bounded regular domain of $\mathbb{R}^N(N \geq 3)$ with smooth boundary $\partial\Omega$, $\mu \in \mathbb{R}, \alpha, \beta, \alpha', \beta'$ are positive constants such that $\alpha+\beta = \frac{4}{N-2}$ and $0 \leq \alpha'+\beta' <$ 4

$N-2.$

In the scalar case , the problem

$$\begin{aligned} -\Delta u &= u^p + \mu u^q \quad \text{in}\Omega \\ u &> 0 \quad \text{in}\Omega \\ u &= 0 \quad \text{on}\partial\Omega, \end{aligned} \tag{1.2}$$

has been considered by several authors . The paper of Brezis - Nirenberg [7] has drawn our attention .

In [7] , they have obtained the following results : Suppose that Ω is a bounded domain in $\mathbb{R}^N, N \geq 3, p = \frac{N+2}{N-2}{}_, q = 1$ and let $\lambda_1 > 0$ denote the first eigenvalue of the operator $-\Delta$ with homogeneous Dirichlet boundary conditions .

(1) If $N \geq 4$, then for any $\mu \in (0, \lambda_1)$ there exists a solution of (1 . 2) .

(2) If $N = 3$, there exists $\mu^* \in (0, \lambda_1)$ such that for any $\mu \in (\mu^*, \lambda_1)$ problem (1 . 2) admits a solution .

2000 *Mathematics Subject Classification* . 35 J 20 , 35 J 50 , 35 J 60 .
Key words and phrases . Elliptic system ; critical Sobolev exponent ; variational method ; moutain pass theorem .

Submitted July 6 , 2004 . Published November 25 , 2004 .

(3) If $N = 3$ and Ω is a ball , then $\mu^* = \frac{\lambda_1}{4}$ and for $\mu \leq \frac{\lambda_1}{4}$ problem (1 . 2) has no solution .

They have also obtained the following results for $1 < q < \frac{N+2}{N-2}$:

(a) There is no solutions of (1 . 2) when $\mu \leq 0$ and Ω is a starshaped domain .

(b) When $N \geq 4, (1.2)$ has at least one solution for every $\mu > 0$.

(c) When $N = 3$, We distinguish two cases :

(i) If $3 < q < 5$, then for every $\mu > 0$ there is a solution of (1 . 2) .

(i i) If $1 < q \leq 3$, then for every μ large enough there is a solution of (1 . 2) . Moreover , (1 . 2) has no solution for every small $\mu > 0$ when Ω is strictly starshaped .

In the vectorial case , Alves et al . [1] and Bouchekif and Nasri [4] have extended the results of [7] to elliptic system . A number of works contributed to study the elliptic system for example : Boccardo and de Figueiredo [3] , de Th é lin and V é lin [1 1] and Conti et al . [8] .

Our aim is to generalize the results of [7] to an elliptic system when the lower order perturbation of $u^{\alpha+1}v^{\beta+1}$ for each equation is weakly coupled i . e .

$$\overset{\rightarrow}{}$$
$$-\Delta U = \nabla H + \mu \nabla G,$$

where

$$\rightarrow \Delta = \begin{pmatrix} \Delta \\ \Delta \end{pmatrix}, \quad H(u,v) = u^{\alpha+1}v^{\beta+1}, \quad U = \begin{pmatrix} u \\ v \end{pmatrix},$$

$G(u,v) = u^{\alpha'+1}v^{\beta'+1}$ and μ is a real parameter .

Our main results are stated as follows :

Theorem 1 . 1 . *If $\alpha+\beta = \frac{4}{N-2}; 0 \leq \alpha'+\beta' < \frac{4}{N-2}; \mu \leq 0$ and Ω is a s tarshaped domain , then (1 . 1) has no s o lution .*

Theorem 1 . 2 . *We suppose that $N \geq 4$ and $\alpha+\beta = \frac{4}{N-2}$. We have :*

- *If $0 < \alpha'+\beta' < \frac{4}{N-2}$, then for every $\mu > 0$ problem (1 . 1) has at least one s o lution .*
- *If $\alpha'+\beta' = 0$, then for every $0 < \mu < \lambda_1$ problem (1 . 1) has a s o lution .*

Theorem 1 . 3 . *Assume that $N = 3$ and $\alpha+\beta = 4$. We distinguish two cas es :*

- *If $2 < \alpha'+\beta' < 4$, then for every $\mu > 0$ problem (1 . 1) has a s o lution .*
- *If $0 < \alpha'+\beta' \leq 2$, then for every μ large enough there exists a s o lutio n to problem (1 . 1) .*

The paper is organized as follows . Section 2 contains some preliminaries and notations . Section 3 contains the proof of nonexistence result . Section 4 deals with the existence theorems proofs .

2 . Preliminaries

Lemma 2 . 1 (Pohozaev identity [1 0]) **.** *Suppose that $(u,v) \in [C^2(\Omega)]^2$ is th e s o lution to th e problem*

$$-\Delta u = \frac{\partial F}{\partial u}(u,v) \quad in \Omega$$
$$-\Delta v = \frac{\partial F}{\partial v}(u,v) \quad in \Omega$$
$$u = v = 0 \quad on \partial\Omega,$$

where Ω *is a bounded domain in* $\mathbb{R}^N$ $(N \geq 3)$ *with smooth boundary* $\partial\Omega$, $F \in C^1(\mathbb{R}^2), F(0,0) = 0$, *then we have*

$$\int_{\partial\Omega} (| \frac{\partial u}{\partial \nu} |^2 + | \frac{\partial v}{\partial \nu} |^2) x\nu d\sigma + (N-2)[\int_\Omega (u \frac{\partial F}{\partial u} + v \frac{\partial F}{\partial v}) dx] = 2N \int_\Omega F(u,v) dx \quad (2.1)$$

where ν *denotes th e exterior unit normal .*

We shall use the following version of the Brezis - Lieb lemma [6] . **Lemma 2 . 2 .** *Assume that* $F \in C^1(\mathbb{R}^N)$ *with* $F(0) = 0$ *and* $| \frac{\partial F}{\partial u_i} | \leq C | u |^{p-1}$. *Let* $(u_n) \subset L^p(\Omega)$ *with* $1 \leq p < \infty$. *If* (u_n) *is bounded in* $L^p(\Omega)$ *and* $u_n \to u$ *a . e . on* Ω, *then*

$$\lim_{n\to\infty} \int_\Omega F(u_n) - F(u_n - u)) = \int_\Omega F(u).$$

Let us define :

$$S_{\alpha+\beta+2} = S_{\alpha+\beta+2}(\Omega) := u \in 0_H^{\inf_1}(\Omega) \setminus \{0\} \frac{\int_\Omega | \nabla u |^2 dx}{(\int_\Omega | u |^{\alpha+\beta+2} dx)^{\frac{2}{\alpha+\beta+2}}}$$

$$S_{\alpha,\beta} = S_{\alpha,\beta}(\Omega) := (u,v) \in [0_H^{\inf_1}(\Omega)]^2 \setminus \{(0,0)\} \frac{\int_\Omega (| \nabla u |^2 + | \nabla v |^2) dx}{(\int_\Omega | u |^{\alpha+1} | v |^{\beta+1} dx)^{\frac{2}{\alpha+\beta+2}}}.$$

Lemma 2 . 3 ([1]) **.** *Let* Ω *be a domain in* $\mathbb{R}^N$(*not necessarily bounded) and* $\alpha + \beta \leq \frac{4}{N-2}$, *then we have*

$$S_{\alpha,\beta} = [(\frac{\alpha+1}{\beta+1})^{\frac{\beta+1}{\alpha+\beta+2}} + (\frac{\alpha+1}{\beta+1})^{\frac{-\alpha-1}{\alpha+\beta+2}}] S_{\alpha+\beta+2}.$$

Moreover , if $S_{\alpha+\beta+2}$ *is attained at* ω_0, *th en* $S_{\alpha,\beta}$ *is attain ed at* $(A\omega_0, B\omega_0)$ *for any real constants* A *and* B *such that* $\frac{A}{B} = (\frac{\alpha+1}{\beta+1})^{1/2}$.

We adopt the following notation :

- For $p > 1$, $\| u \|_p = [\int_\Omega | u |^p dx]^{\frac{1}{p}}$;
- $H_0^1(\Omega)$ is the Sobolev space endowed with the norm $\| u \|_{1,2} = [\int_\Omega | \nabla u |^2 dx]^{1/2}$;
- $\| (u,v) \|^2_E := \| u \|^2_{1,2} + \| v \|^2_{1,2}$;
- $E := [H_0^1(\Omega)]^2$;
- E' denotes the dual of E;
- $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent ;
- $u^+ := \max(u,0)$ and $u^- = u^+ - u$.

The functional associated to problem (1 . 1) is written as

$$J(u,v) := \frac{1}{2} \| (u,v) \|^2_E - \int_\Omega (u^+)^{\alpha+1} (v^+)^{\beta+1} dx - \mu \int_\Omega (u^+)^{\alpha'+1} (v^+)^{\beta'+1} dx. \quad (2.2)$$

3 . Nonexistence result

Theorem 1 . 1 is a direct consequence of the Pohozaev identity . *Proof of Theorem 1 . 1 .* Arguing by contradiction . Suppose that problem (1 . 1) has a solution $(u, v) \neq (0, 0)$, applying Lemma 2 . 1 and putting

$$F(u, v) = H(u, v) + \mu G(u, v),$$

 the expression (2 . 1) becomes

$$\int_{\partial\Omega}(|\frac{\partial u}{\partial\nu}|^2+|\frac{\partial v}{\partial\nu}|^2)x\nu d\sigma=\mu[2N-(N-2)(\alpha'+\beta'+2)]\int_{\Omega}|u|^{\alpha'+1}|v|^{\beta'+1}dx.$$

Since $2N-(N-2)(\alpha'+\beta'+2)>0$ and the fact that Ω is st arshaped with respect to the origin , we get

$$0\leq\int_{\partial\Omega}(|\frac{\partial u}{\partial\nu}|^2+|\frac{\partial v}{\partial\nu}|^2)x\nu d\sigma<0.$$

A contradiction . Hence (1 . 1) has no a solution for $\mu\leq 0$. □

4 . Existence results

The proof of Theorems 1 . 2 and 1 . 3 are based on the following Ambrosetti - Rabinowitz result [2] .

Lemma 4 . 1 (Mountain Pass Theorem) **.** *Let J be a C^1 functional on a Banach space E. Suppose there exits a neighborhood V of 0 in E and a positive constant ρ such that*

(i) *$J(u,v)\geq\rho$ for every U in th e boundary of V.*

(i i) *$J(0,0)<\rho$ and $J(\varphi,\psi)<0$ for s ome $\Psi:=(\varphi,\psi) element-slash V$. We s e t*

$$c=\inf_{\phi\in\Gamma}\max_{t}{}_{\in[0,1]}J(\phi(t))$$

with $\Gamma=\{\phi\in C([0,1],E) \ :\phi(0)=0,\phi(1)=\Psi\}$. Then there exists a s equence (u_n,v_n) in E such that $J(u_n,v_n)\to c$ and $J'(u_n,v_n)\to 0$ in E'. *Proof .* Using Holder ' s inequality and Sobolev injection , we obtain that

$$\begin{aligned}J(u,v)&=\frac{1}{2}\|(u,v)\|_E^2-\int_{\Omega}(u^+)^{\alpha+1}(v^+)^{\beta+1}dx-\mu\int_{\Omega}(u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx\\&\geq\frac{1}{2}\|(u,v)\|_E^2-A\|(u,v)\|2^*E-B\|(u,v)\|_E^{\alpha'+\beta'+2}\end{aligned}$$

where A and B are positive constants .

If $\alpha'+\beta'>0$ then (i) is satisfying for small norm $\|(u,v)\|E=R$. If $\alpha'+\beta'=0$, we have

$$J(u,v)\geq\frac{1}{2}(1-\frac{\mu}{\lambda_1})\|(u,v)\|2E-A\|(u,v)\|_E^{2^*}$$

and condition (i) is still satisfied for $\mu<\lambda_1$ and $R<(\frac{1-\frac{\mu}{\lambda_1}}{2A})\frac{1}{2^*-2}$. For any $(\varphi,\psi)\in E$ with $\varphi\neq 0$ and $\psi\neq 0$, we have that $\lim_{t\to+\infty}J(t\varphi,t\psi)=-\infty$. Thus , there are many (φ,ψ) satisfying (ii). It will be important to use with a special $(\varphi,\psi):=(t_0\varphi 0,t_0\psi 0)$ for some $t_0>0$ chosen large enough so that $(\varphi,\psi) element-slash V, J(\varphi,\psi)<0$ and $\sup_{t\geq 0}J(t\varphi,t\psi)<\frac{2^*}{N}(\frac{S_{\alpha,\beta}}{2^*})^{N/2}$. Then there exists a sequence $(u_n,v_n)\in E$ such that $J(u_n,v_n)\to c$ and $J'(u_n,v_n)\to 0$ in E'. □

Lemma 4 . 2 . *Suppose $\mu>0$ and le t (u_n,v_n) be a s equence in E such that $J(u_n,v_n)\to c$ and $J'(u_n,v_n)\to 0$ in E' with*

$$c<\frac{2^*}{N}(\frac{S_{\alpha,\beta}}{2*})^{N/2}=\frac{2}{N-2}(\frac{S_{\alpha,\beta}}{2*})^{N/2}$$

Then (u_n,v_n) is relatively compact in E.

Proof . We show that the sequence (u_n, v_n) is bounded in E. Since (u_n, v_n) satisfies :

$$\frac{1}{2} \parallel (u_n, v_n) \parallel 2E - \int_\Omega (n_u^+)^{\alpha+1}(n_v^+)^{\beta+1}dx - \mu \int_\Omega (n_u^+)^{\alpha'+1}(n_v^+)^{\beta'+1}dx = c + o(1) \tag{4.1}$$

and

$$\parallel (u_n, v_n) \parallel_E^2 -2^* \int_\Omega (n_u^+)^{\alpha+1}(n_v^+)^{\beta+1}dx - \mu(\alpha' + \beta' + 2) \int_\Omega (n_u^+)^{\alpha'+1}(n_v^+)^{\beta'+1}dx = \langle \varepsilon_n, (u_n, v_n)\rangle$$

(4 . 2) with $\varepsilon_n \to 0$ in E'. Combining (4 . 1) and (4 . 2) , we obtain

$$(\frac{2^*}{2} - 1) \int_\Omega (n_u^+)^{\alpha+1}(n_v^+)^{\beta+1}dx + \mu(\frac{\alpha' + \beta'}{2}) \int_\Omega (n_u^+)^{\alpha'+1}(n_v^+)^{\beta'+1}dx \leq c + o(1) + \parallel \varepsilon_n \parallel E' \parallel (u_n, v_n) \parallel E. \tag{4.3}$$

From this inequality , we obtain

$$\int_\Omega (n_u^+)^{\alpha+1}(n_v^+)^{\beta+1}dx \leq C,$$
$$\int_\Omega (n_u^+)^{\alpha'+1}(n_v^+)^{\beta'+1}dx \leq C.$$

Where C is any generic positive constant . Therefore , the sequence (u_n, v_n) is bounded in E. By the Sobolev embedding Theorem , there exists a subsequence again denoted by (u_n, v_n) such that

- $(u_n, v_n) \to (u, v)$weaklyin$E$
- $(u_n, v_n) \to (u, v)$ strongly in $L^r \times L^q$ for $2 \leq r, q < 2^*$
- $(u_n, v_n) \to (u, v)$ a . e . on Ω.

Since $w_n := u_n^\alpha n_v^{\beta+1}$ and $t_n := u_n^{\alpha+1} n_v^\beta$ are bounded sequences in $[L_{\frac{2^*}{2^*-1}}(\Omega)]^2$, these sequences converge to $w := u^\alpha v^{\beta+1}$ and to $t := u^{\alpha+1}v^\beta$ respectively . Passing to the limit , we obtain

$$-\Delta u = (\alpha + 1)(u^+)^\alpha (v^+)^{\beta+1} + \mu(\alpha' + 1)(u^+)^{\alpha'}(v^+)^{\beta'+1}$$
$$-\Delta v = (\beta + 1)(u^+)^{\alpha+1}(v^+)^\beta + \mu(\beta' + 1)(u^+)^{\alpha'+1}(v^+)^{\beta'}$$

i . e

$$\parallel (u, v) \parallel 2E = 2^* \int_\Omega (u^+)^{\alpha+1}(v^+)^{\beta+1}dx + \mu(\alpha' + \beta' + 2) \int_\Omega (u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx$$

Moreover ,

$$J(u, v) = (\frac{2^*}{2} - 1) \int_\Omega (u^+)^{\alpha+1}(v^+)^{\beta+1}dx + \mu(\frac{\alpha' + \beta'}{2}) \int_\Omega (u^+)^{\alpha'+1}(v^+)^{\beta'+1}dx \geq 0.$$

We put

$$u = u_n + \varphi n, \quad v = v_n + \psi_n \quad \text{and} \quad H(u_n, v_n) = u_n^{\alpha+1} n_v^{\beta+1}$$

Applying Lemma 2 . 2 for $H(u_n, v_n)$ and the following two relations (Brezis - Lieb [6])

$$\| u_n \| 2 = \| u - \varphi n \| 2 = \| u \|^2 + \| \varphi n \|^2 + o(1),$$
$$\| v_n \| 2 = \| v - \varphi n \| 2 = \| v \|^2 + \| \psi_n \|^2 + o(1),$$

 we obtain

$$J(u,v)+\frac{1}{2}\parallel(\varphi n,\psi_n)\parallel 2E-\int_{\Omega}H(\varphi_n^+,\psi_n^+)dx=c+o(1) \tag{4.4}$$

and

$$\parallel(\varphi n,\psi_n)\parallel_E^2+\parallel(u,v)\parallel_E^2=2^*_{+}{}^{[\int_\Omega(H\ (u^+,}_{[\mu\quad(\alpha'+\quad\beta'+}v^+2^{)}_{)}{}^{+H(\varphi_n^+,\ \psi_n^+))dx]}_{\int_\Omega(u+)\alpha'+1(v+)^{\beta'+1}dx}+o(1). \tag{4.5}$$

From this equality , we deduce

$$\parallel(\varphi n,\psi_n)\parallel 2E=2^*\int_{\Omega}H(\varphi_n^+,\psi_n^+)dx+o(1).$$

We may therefore assume that

$$\parallel(\varphi n,\psi_n)\parallel 2E\to k\quad\text{and}\quad 2^*\int_{\Omega}H(\varphi_n^+,\psi_n^+)dx\to k.$$

By the Sobolev inequality ,

$$\parallel(\varphi n,\psi_n)\parallel_E^2\geq S_{\alpha,\beta}(\int_{\Omega}(\varphi_n^+)^{\alpha+1}(\psi_n^+)^{\beta+1}dx)\frac{2}{2^*}.$$

In the limit , $k\geq S_{\alpha,\beta}(\frac{k}{2*})^{2/2^*}$. It follows that either $k=0$ or $k\geq 2^*(\frac{S_{\alpha,\beta}}{2*})^{N/2}$.

We show that $(u_n,v_n)\to(u,v)$ strongly in E i . e . $(\varphi n,\psi_n)\to(0,0)$ strongly in E. Suppose that $k\geq 2^*(\frac{S_{\alpha,\beta}}{2*})^{N/2}$. Since

$$J(u,v)+\frac{k}{N}=c$$

and $J(u,v)\quad\geq 0$, then $\frac{k}{N}\quad\leq c$ i . e . $c\geq\quad\frac{2^*}{N}(\frac{S_{\alpha,\beta}(\Omega)}{2^*})^{N/2}$ in contradiction with the hypothesis . Thus $k=0$ and $(u_n,v_n)\to(u,v)$ strongly in E. $\square$

Proof of Theorem 1 . 2 . It suffices to apply the mountain pass theorem with the value $c<\frac{2^*}{N}(\frac{S_{\alpha,\beta}(\Omega)}{2*})^{N/2}$. We have to show that this geometric condition on c is satisfied . Following the method in [7] . Without loss of generality we assume that
$0\in\Omega$, we use the test function

$$\omega_\varepsilon(x)=\frac{\varphi(x)}{(\varepsilon+\mid x\mid^2)\frac{N-2}{2}},\quad\varepsilon>0$$

where φ is a cut - off positive function such that $\varphi\equiv 1$ in a neighborhood of 0 . Let A and B be positive constants such that

$$\frac{A}{B}=(\frac{\alpha+1}{\beta+1})^{1/2}$$

then $(A\omega_\varepsilon,B\omega_\varepsilon)$ is a solution of

$$\begin{aligned}-\Delta u&=(\alpha+1)u^\alpha v^{\beta+1}\quad\text{in}\mathbb{R}^N\\ -\Delta v&=(\beta+1)u^{\alpha+1}v^\beta\quad\text{in}\mathbb{R}^N\\ u(x)=0,\quad v(x)&=0\quad\text{as}\mid x\mid\to+\infty\end{aligned}$$

By [7 , lemma 1] , we obtain

$$\sup_{t\geq 0}J(tA\omega_\varepsilon,tB\omega_\varepsilon)\leq\frac{2^*}{N}(\frac{S_{\alpha,\beta}}{2*}^{)^{N/2}}+O(\varepsilon\frac{N-2}{2})-\mu K\varepsilon^\theta$$

where K is a positive constant independent of ε, and $\theta := (4-(\alpha'+\beta')(N-2))/4$.

For $\theta < \frac{N-2}{2}$ if $N > 4$ the inequality is satisfying for all $0 \leq \alpha'+\beta' < \frac{4}{N-2}$. Thus we obtain

$$\sup_{t\geq 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) < \tfrac{2^*}{N}\big(\tfrac{S_{\alpha,\beta}}{2*}\big)^{N/2} \quad \text{for } \varepsilon > 0 \text{ small enough .}$$

Then problem (1 . 1) has a solution for every $\mu > 0$.

For $N = 4$, we distinguish two cases . Case 1 : We have $\theta < 1$ for all $\alpha'+\beta' > 0$. Case 2 : If $\alpha'+\beta' = 0$, we obtain

$$\sup_{t\geq 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) \leq (\frac{S_{\alpha,\beta}}{4})^2 + O(\varepsilon) - \mu K\varepsilon \mid \log\varepsilon \mid,$$

so for $\varepsilon > 0$ small enough , $\sup_{t\geq 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) < (\frac{S_{\alpha,\beta}}{4})^2$.

Note that the maximum principle ensures the positivity of solution . □ *Proof of Theorem 1 . 3 .* In three dimension the situation is different . We have

$$\sup_{t\geq 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) \leq 2(\frac{S_{\alpha,\beta}}{6})^{3/2} + O(\varepsilon^{1/2}) - \mu K\varepsilon^\theta.$$

In this case we distinguish two cases .

$$\text{(i)} \quad 0 < \theta < \frac{1}{2} \text{if} 2 < \alpha'+\beta' < 4,$$

$$\text{(i i)} \quad \theta \geq \tfrac{1}{2} \text{ if } 0 < \alpha'+\beta' \leq 2.$$

In case (i) we have the same conclusion as in the previous proof for $(N \geq 4)$. So for the case $0 < \alpha'+\beta' \leq 2$, the existence of positive solution is assured for μ large enough . It follows that $\sup_{t\geq 0} J(tA\omega_\varepsilon, tB\omega_\varepsilon) < 2(\frac{S_{\alpha,\beta}}{6})^{3/2}$. Thus (1 . 1) has a solution . □

References

[1] C . O . Alves , D . C . de Morais Filho and M . A . S . Souto ; *On Systems of El liptic equations involving subcritical or critical Sobolev exponents* , Nonlinear Anal . N . 5 , Ser . Theory Methods , 42 (2000) , 771 – 787 .

[2] A . Ambrosetti and P . H . Rabinowitz ; *Dual variational methods in critical point theory and applications* , J . Funct . Anal . , 14 (1 973) , 349 - 381 .

[3] L . Boccardo , D . G . de Figueiredo : *Some remarks on system of quasilinear el lip tic equations* , Nonlinear diff . eq . appl . 9 (2002) , pp . 309 - 323 .

[4] M . Bouchekif and Y . Nasri ; *On a class of el liptic system involving critical Sobolev exponent* , Preprint Tlemcen 2004 .

[5] H . Brezis ; *Some Variational Problems with Lack of Compactness* , Proceedings of Symposia in Pure Mathematics , Vol . 45 (1 986) , 1 65 - 20 1 .

[6] H . Brezis and E . Lieb ; *A relation between point wise convergence of functions and convergence of functionals* , Proc . A . M . S . Vol . 48 , No . 3 (1 993) , 486 - 499 .

[7] H . Brezis and L . Nirenberg ; *Positive Solutions of Nonlinear El liptic Equations Involving Critical Exponents* , Comm . Pure Appl . Math , Vol . 36 (1 983) , 437 - 477 .

[8] M . Conti , L . Merizzi and S . Terracini ; *On the existence of many solutions for a class of superlinear el liptic system* , J . Diff . Eq . 1 67 (2000) , 357 – 387 .

[9] S . I . Pohozaev ; *Eigenfunctions of the equation* $\Delta u + \lambda f(u) = 0$, Nonlinearity Doklady Akad . Nauk SSRR 1 65 , (1 965) , 9 - 36 . [10] P . Pucci and J . Serrin ; *A general variational id entity* , Indiana University Mathematics Jour - nal , (1 986) , 68 1 - 703 . [1 1] F . de Th é lin and J . V é lin ; *Existence and nonexistence of nontrivial solutions for some nonlin - ear el l ip tic systems* , Revista Mathematica Universidad complutense de Madrid , vol . 6 (1 993) , 1 53 - 1 93 .

Mohammed Bouchekif Departement of Mathematics , University of Tlemcen B . P . 1 1 9 Tlemcen 1 300 0 , Algeria

E - mail address : m_______ bouchekif @ mai l . univ - t lemcen . dz

Yasmina Nasri Departement of Mathematics , University of Tlemcen B . P . 1 1 9 Tlemcen 1 300 0 , Algeria

E - mail address : $\mathtt{y}^{line-n}$ asri @ mail .n — u iv - tlemcen . dz