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Solutions of the Cahn-Hilliard Equation with Time-and Space-Fractional Derivatives

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Abstract: In this paper, by introducing the fractional derivative in the sense of Caputo, we apply the Adomian decomposition method for Cahn-Hilliard with time-and space-fractional derivative. As a result, numerical solutions are obtained in a form of rapidly convergent series with easily computable components.

Keywords: Caputo Fractional derivative; Adomian method; Cahn-Hilliard equation; Evolution Equations.

1 Introduction

Since the introduction by Adomian of the decomposition method [4, 5] at the begin of 1980s, the algorithm has been widely used for obtaining analytic solutions of physically significant equations [4–6, 20, 30–35]. With this method, we can easily obtain approximate solutions in the form of a rapidly convergent infinite series with each term computed conveniently[1, 2, 10, 17].

As it is known, for the nonlinear equations with derivatives of integer order, many methods are used to derive approximation solutions [3, 9, 14, 22, 27, 29]. However, for the fractional differential equations, there are only limited approaches, such as Laplace transform method [24], the Fourier transform method [21], the iteration method [25] and the operational method [23].
In recent years, considerable interest in fractional differential equations has been stimulated due to their numerous applications in the area of physics and engineering [11, 18, 36], like phenomena in electromagnetic theory, acoustics, electrochemistry and material science [12, 16, 24, 25, 36]. In [28], the Adomian decomposition method (ADM) is applied to the Cahn-Hilliard equation

$$u_t = \gamma u_x + 6u(u_x)^2 + (3u^2-1)u_{xx} - u_{xxxx},\ \ \gamma \geq 0. \tag{1}$$

This equation is related with a number of interesting physical phenomena like the spinodal decomposition, phase separation and phase odering dynamics. It is also very crucial in material sciences [7, 8, 15]. On the other hand this equation is very hard and difficult to solve. Many articles have investigated mathematically and numerically this equation [13, 19].

The aim of this paper is to use the ADM method to study the Cahn-Hilliard equation with time-and space-fractional derivatives of this form

$$D_t^{\alpha} u - \gamma D_x^{\beta} u - 6u(D_x^{\beta} u)^2 - (3u^2-1)u_{xx} + u_{xxxx} = 0. \tag{2}$$

Here α and β are the parameters standing for the order of the fractional time and space derivatives, respectively and they satisfy $0 < \alpha \leq 1$, $0 < \beta \leq 1$ and $x > 0$. In fact, different response equations can be

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obtained when at lest one of the parameters varies. When $\alpha = \beta = 1$, the fractional equation reduces to the Cahn-Hilliard equation (1).

We introduce Caputo fractional derivative and apply the ADM to derive numerical solutions of the equation (2).

The paper is organized as follows. In SecII, some necessary details on the fractional calculus are provided. In SecIII, the Cahn-Hilliard equation with time and space-fractional derivative is studied with the ADM. Finally, conclusions follow.

2 Description of Fractional Calculus

There are several mathematical definitions about fractional derivative [24, 25]. Here, we adopt the two usually used definitions: the Caputo and its reverse operator Riemann-Liouville. That is because Caputo fractional derivative allows traditional initial condition assumption and boundary conditions. More details one can consult [24]. In the following, we will give the necessary notation and basic definitions.

Definition 1 *A real valued function $f(x), x > 0$ is said to be in the space $C_\mu, \mu \in IR$ if there exists a real number $p > \mu$ such that $f(x) = x^p f_1(x)$ where $f_1(x) \in C([0, \infty))$.*

Definition 2 *A function $f(x), x > 0$ is said to be in the space $C^n_\mu, n \in IR$, if $f^{(n)} \in C_\mu$.*

Definition 3 *The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, for a function $f \in C_\mu, (\mu \geq -1)$ is defined as*

$$\begin{array}{c} J^\alpha f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)dt; \quad \alpha > 0, x > 0 \\ J^0 f(x) = f(x). \end{array} \tag{3}$$

For the convenience of establishing the results for the Cahn-Hilliard equation, we give one basic property:

$$J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x). \tag{4}$$

For the expression (3), when $f(x) = x^\beta$ we get another expression that will be used later:

$$J^\alpha x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)} x^{\alpha+\beta}. \tag{5}$$

Definition 4 *The fractional derivative of $f \in C^n_{-1}$ in the Caputo's sense is defined as*

$$D^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)d\tau, & n-1 < \alpha < n, n \in IN^*, \\ \frac{d^n}{dt^n} f(t), \alpha = n. \end{cases} \tag{6}$$

According to the Caputo's derivative, we can easily obtain the following expressions:

$$D^\alpha K = 0; \ K \text{ is a constant.}$$

$$D^\alpha t^\beta = \begin{cases} \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)} t^{\beta-\alpha}, \beta > \alpha - 1, \\ 0, & \beta \leq \alpha - 1. \end{cases} \tag{7}$$

Details on Caputo's derivative can be found in [24].

Remark 1 *In this paper, we consider equation (2) (with time-and space fractional derivatives). When $\alpha \in IR^+$, we have:*

$$D_t^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} \frac{\partial^n u(x,\tau)}{\partial \tau^n} d\tau, n-1 < \alpha < n \\ \frac{\partial^n u(x,t)}{\partial t^n}, \alpha = n. \end{cases} \tag{8}$$

The form of the space fractional derivative is similar to the above and we just omit it here.

3 Applications of the ADM Method

Consider the Cahn-Hilliard equation with time and space-fractional derivatives Eq.(2). In order to solve numerical solutions for this equation by using ADM method, we rewrite it in the operator form:

$$D_t^{\alpha}u = \gamma D_x^{\beta}u + 6u(D_x^{\beta}u)^2 + (3u^2-1)u_{xx} - u_{xxxx}, \quad \beta>0, \alpha>0, \gamma>0, \tag{9}$$

where the operators D_t^{α} and D_x^{β} stand for the fractional derivative and are defined as in (6).
Take the initial condition as

$$u(x,0) = f(x). \tag{10}$$

Applying the operator J^{α}, the inverse of D^{α} on corresponding sub-equation of Eq.(9), using the initial condition (10), yields:

$$u(x,t) = f(x) + \gamma J^{\alpha}\Phi_1(u(x,t)) + 6J^{\alpha}\Phi_2(u(x,t)) + J^{\alpha}\Phi_3(u(x,t)) - J^{\alpha}L(u(x,t), \tag{11}$$

where $\Phi_1(u) = D_x^{\beta}u,\ \ \Phi_2(u) = u(D_x^{\beta}u)^2,\ \ \Phi_3(u) = (3u^2-1)u_{xx},\quad L(u(x,t)) = u_{xxxx}$.

Following Adomian decomposition method [4, 5], the solution is represented as infinite series like

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{12}$$

The nonlinear operators $\Phi_1(u)$, $\Phi_2(u)$ and $\Phi_3(u)$ are decomposed in these forms

$$\Phi_1(u) = \sum_{n=0}^{\infty} A_n, \quad \Phi_2(u) = \sum_{n=0}^{\infty} B_n, \quad \Phi_3(u) = \sum_{n=0}^{\infty} C_n, \tag{13}$$

where A_n, B_n , C_n are the so-called Adomian polynomials and have the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\Phi_1\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0} = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[D_x^{\beta}\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0}, \tag{14}$$

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\Phi_2\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0} = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\left(D_x^{\beta}\sum_{k=0}^{\infty}\lambda^k u_k\right)^2\right]_{\lambda=0}, \tag{15}$$

$$C_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\left(3\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)^2 - 1\right)\left(\sum_{k=0}^{\infty}\lambda^k u_{kxx}\right)\right]_{\lambda=0}.$$

In fact, these Adomian polynomials can be easily calculated. Here we give the first three components of these polynomials:

$$\begin{gathered} A_0 = D_x^{\beta}u_0, A_1 = D_x^{\beta}u_1, \\ A_2 = D_x^{\beta}u_2, A_3 = D_x^{\beta}u_3. \end{gathered} \tag{16}$$

The first three components of B_n are

$$\begin{gathered} B_0 = u_0(D_x^{\beta}u_0)^2, B_1 = 2u_0 D_x^{\beta}u_0 D_x^{\beta}u_1 + (D_x^{\beta}u_0)^2 u_1, \\ B_2 = 4u_0 D_x^{\beta}u_0 D_x^{\beta}u_2 + 2u_0(D_x^{\beta}u_1)^2 + 3u_1 D_x^{\beta}u_0 D_x^{\beta}u_1 + u_2(D_x^{\beta}u_0)^2, \\ B_3 = \tfrac{1}{3}(12u_0 D_x^{\beta}u_1 D_x^{\beta}u_2 + 5u_1(D_x^{\beta}u_1)^2 + 10u_1 D_x^{\beta}u_0 D_x^{\beta}u_2 + \\ +12u_0 D_x^{\beta}u_0 D_x^{\beta}u_3 + 6u_2 D_x^{\beta}u_0 D_x^{\beta}u_1 + 2u_1u_2 D_x^{\beta}u_0 + 3u_3(D_x^{\beta}u_0)^2). \end{gathered} \tag{17}$$

and those of C_n are given by:

$$\begin{gathered} C_0 = 3u_0^2u_{0xx} - u_{0xx}, C_1 = 3(2u_0u_1u_{0xx} + u_0^2u_{1xx}) - u_{1xx}, \\ C_2 = 3(u_1^2u_{0xx} + 2u_0u_2u_{0xx} + 2u_0u_1u_{1xx} + u_0^2u_{2xx}) - u_{2xx}, \\ C_3 = 3((2u_1u_2 + 2u_0u_3)u_{0xx} + (u_1^2 + 2u_0u_2)u_{1xx} + 2u_0u_1u_{2xx} + u_0^2u_{3xx}) - u_{3xx}. \end{gathered} \tag{18}$$

Other polynomials can be generated in a like manner. Substituting the decomposition series (12) and (13) into Eq.(11), yields the following recursive formula:

$$u_0(x,t) = f(x),\ u_{n+1}(x,t) = \gamma J^{\alpha}(A_n) + 6J^{\alpha}(B_n) + J^{\alpha}(C_n) - J^{\alpha}(L_x(u_n));\ n \geq 0. \tag{19}$$

The Adomian decomposition method converges generally very quickly. Details about its convergence and convergence speed can be found in [1, 2, 10, 17]. Here, according to the above steps, we will derive the numerical solution for the Eq.(9) in details.

3.1 Numerical Solutions of Time-Fractional Cahn-Hilliard Equation

Consider the following form of the time-fractional equation (for $\gamma = 1$)

$$D_t^{\alpha} u = u_x + 6u(u_x)^2 + (3u^2 - 1)u_{xx} - u_{xxxx}, \tag{20}$$

with the initial condition [28]

$$u(x,0) = f(x) = \tanh(\frac{\sqrt{2}}{2}x), \tag{21}$$

The exact solution of (20) for the special case $\alpha = \beta = 1$ is :

$$u(x,t) = \tanh(\frac{\sqrt{2}}{2}(x+t)). \tag{22}$$

In order to obtain numerical solution of equation (20), substituting the initial condition (21) and using the Adomian polynomials (15-18) into the expression (19), we can compute the results. For simplicity, we only give the first few terms of series:

$$\begin{aligned}
u_0 &= \tanh(\frac{\sqrt{2}}{2}x),\\
u_1 &= J^{\alpha}(A_0) + 6J^{\alpha}(B_0) + J^{\alpha}(C_0) - J^{\alpha}(u_{0xxxx})\\
&= J^{\alpha}(u_{0x}) + 6J^{\alpha}(u_0 u_{0x}^2) + J^{\alpha}(3u_0^2 u_{0xx} - u_{0xx}) - J^{\alpha}(u_{0xxxx}) = f_1 \frac{t^{\alpha}}{\Gamma(\alpha+1)},\\
u_2 &= J^{\alpha}(A_1) + 6J^{\alpha}(B_1) + J^{\alpha}(C_1) - J^{\alpha}(u_{1xxxx})\\
&= J^{\alpha}(u_{1x}) + 6J^{\alpha}(2u_0 u_{0x} u_{1x} + (u_{0x})^2 u_1) + J^{\alpha}(3(2u_0 u_1 u_{0xx} + u_0^2 u_{1xx}) - u_{1xx}) - J^{\alpha}(u_{1xxxx})\\
&= f_2 \frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
u_3 &= J^{\alpha}(A_2) + 6J^{\alpha}(B_2) + J^{\alpha}(C_2) - J^{\alpha}(u_{2xxxx})\\
&= J^{\alpha}(u_{2x}) + 6J^{\alpha}(4u_0 u_{0x} u_{2x} + 2u_0(u_{1x})^2 + 3u_1 u_{0x} u_{1x} + u_2(u_{0x})^2)\\
&+ J^{\alpha}(3(u_1^2 u_{0xx} + 2u_0 u_2 u_{0xx} + 2u_0 u_1 u_{1xx} + u_0^2 u_{2xx}) - u_{2xx}) - J^{\alpha}(u_{1xxxx})\\
&= f_3 \frac{t^{3\alpha}}{\Gamma(3\alpha+1)},
\end{aligned} \tag{23}$$

where

$$f(x) = u_0 = \tanh(\frac{\sqrt{2}}{2}x), \tag{24}$$

$$f_1(x) = f_x + 6ff_x^2 + (3f^2 f_{xx} - f_{xx}) - f_{xxxx}, \tag{25}$$

$$f_2(x) = f_{1x} + 6(2ff_x f_{1x} + f^2 f_1) + 3(2ff_1 f_{xx} + f^2 f_{1xx}) - f_{1xx} - f_{1xxxx}, \tag{26}$$

$$\begin{aligned}
f_3(x) = f_{2x} &+ 6(4ff_x f_{2x} + 2f(f_{1x})^2 \tfrac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} + 3f_1 f_x f_{1x} \tfrac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} + f_2(f_x)^2) +\\
&+ (3(f_1^2 f_{xx} \tfrac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} + 2ff_2 f_{xx} + 2ff_1 f_{1xx} \tfrac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)} + f^2 f_{2xx}) - f_{2xx}) - f_{1xxxx}.
\end{aligned} \tag{27}$$

Then we have the numerical solution of time-fractional equation (20) under the series form

$$u(x,t) = f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + f_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + ... \tag{28}$$

In order to check the efficiency of the proposed ADM for the equation (20), we draw figures for the numerical solutions with $\alpha = \frac{1}{2}$ as well as the exact solution (22) when $\alpha = \beta = 1$. Figure(a) shows the exact solution. Figure(b) stands for the numerical solution (28). From these figures, we can appreciate how closely are the two solutions. This is to say that good approximations are achieved using the ADM method.

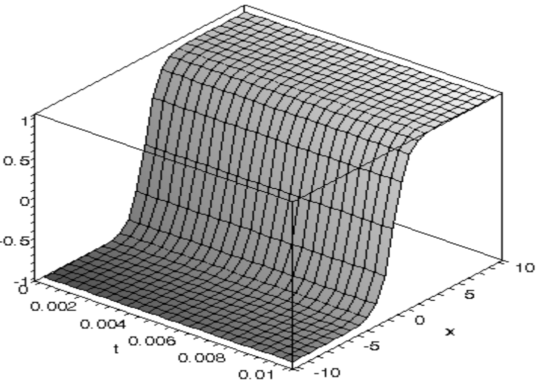

Figure 1: Exact solution of Eq.(20)

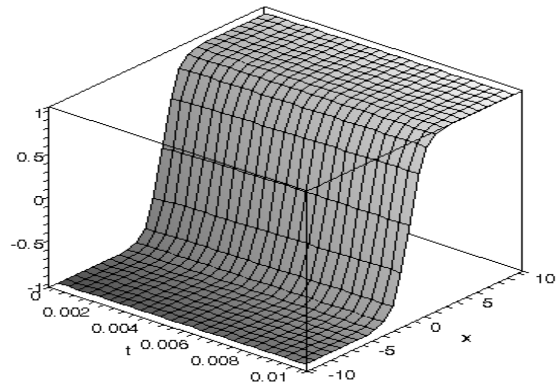

Figure 2: Solution of Eq.(20)obtained by ADM method for $\alpha = \frac{1}{2}$

3.2 Numerical Solutions of Space-Fractional Cahn-Hilliard Equation

In this section, we will take the space-fractional equation as another example to illustrate the efficiency of the method. As the main computation method is the same as the above, we will omit the heavy calculation and only give some necessary expressions.

Considering the operator form of the space-fractional equation (for $\gamma = 1$)

$$u_t = D_x^{\beta}u + (3u^2 - 1)u_{xx} + 6u(D_x^{\beta}u)^2 - u_{xxxx}, \quad 0 < \beta < 1. \tag{29}$$

Assuming the initial condition as

$$u(x,0) = f(x) = x^2. \tag{30}$$

Initial condition has been taken as the above polynomial to avoid heavy calculation of fractional differentiation.

In order to estimate the numerical solution of equation (29), substituting (15-17) and the initial condition (30) into (19), we get the Adomian solution. Here, we give the first few terms of the series solution:

$$\begin{aligned}
u_0 &= f(x) = x^2,\\
u_1 &= J(A_0) + 6J(B_0) + J(C_0) - J(D_0)\\
&= J(D_x^{\beta}u_0) + 6J(u_0(D_x^{\beta}u_0)^2) + J(3u_0^2u_{0xx} - u_{0xx}) - J(u_{0xxxx})\\
&= (f_1x^{2-\beta} + f_2x^{6-2\beta} + 6x^4 - 2)t,\\
u_2 &= J(A_1) + 6J(B_1) + J(C_1) - J(L_x(u_1))\\
&= J(D_x^{\beta}u_1) + 6J\left(2u_0D_x^{\beta}u_0D_x^{\beta}u_1 + (D_x^{\beta}u_0)^2u_1\right) + J\left(3(2u_0u_1u_{0xx} + u_0^2u_{1xx}) - u_{1xx}\right) - J\left(u_{1xxxx}\right)\\
&= \frac{t^2}{2}\left(f_3(x) + 6f_4(x) + f_5(x) - f_6(x)\right)
\end{aligned} \tag{31}$$

where

$$f(x)=x^2, f_1=\frac{\Gamma(3)}{\Gamma(3-\beta)}, f_2=\frac{6\Gamma^2(3)}{\Gamma^2(3-\beta)},$$

$$f_3(x)=f_1\frac{\Gamma(3-\beta)}{\Gamma(3-2\beta)}x^{2-2\beta}+f_2\frac{\Gamma(7-2\beta)}{\Gamma(7-3\beta)}x^{6-3\beta}+6\frac{\Gamma(5)}{\Gamma(5-\beta)}x^{4-\beta} \tag{32}$$

$$\begin{aligned}f_4(x)=2f_1\tfrac{\Gamma(3)}{\Gamma(3-2\beta)}x^{6-3\beta}+2f_2\tfrac{\Gamma(3)\Gamma(7-2\beta)}{\Gamma(3-\beta)\Gamma(7-3\beta)}x^{8-3\beta}\\ +12\tfrac{\Gamma(3)\Gamma(5)}{\Gamma(5-\beta)\Gamma(3-\beta)}x^{6-\beta}+\tfrac{\Gamma^2(3)}{\Gamma^2(3-\beta)}(f_1x^{6-3\beta}+f_2x^{10-4\beta}+x^{8-2\beta}-2x^{4-2\beta}),\end{aligned} \tag{33}$$

$$\begin{aligned}f_5(x)=12f_1x^{4-\beta}+12f_2x^{8-2\beta}+72x^6-24x^2+3(2-\beta)(1-\beta)f_1x^{4-\beta}\\ +3(6-2\beta)(5-2\beta)f_2x^{8-2\beta}+3.72x^6+-(2-\beta)(1-\beta)f_1x^{-\beta}-(6-2\beta)(5-2\beta)f_2x^{4-2\beta}-72x^2,\end{aligned}$$

$$f_6(x)=(2-\beta)(1-\beta)(-\beta)(1-\beta-1)f_1x^{-2-\beta}+(6-2\beta)(5-2\beta)(4-2\beta)(3-2\beta)f_2x^{2-2\beta}. \tag{34}$$

Then we obtain a numerical solution of space-fractional equation (29) in series form

$$u(x,t)=x^2+t\left(f_1x^{2-\beta}+f_2x^{6-2\beta}+6x^4-2\right)+\frac{t^2}{2}\left(f_3(x)+6f_4(x)+f_5(x)-f_6(x)\right)+... \tag{35}$$

Figures(c,d) show, respectively, the numerical solutions given by expression (35) for the equation (29) with $\beta=\frac{1}{2}$ and $\beta=1$. From these figures, we can appreciate the convergence rapidity of Adomian solutions.

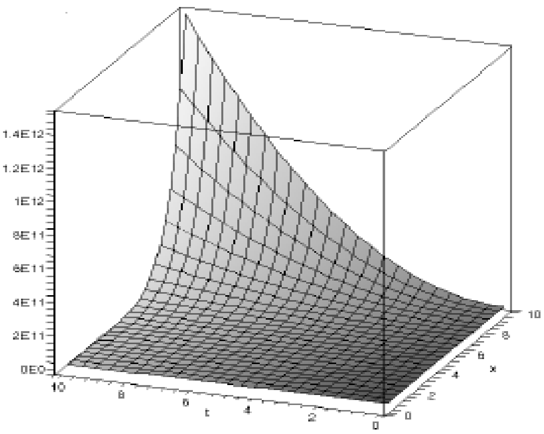

Figure 3: Solution of Eq.(29) obtained by ADM method for $\beta=\frac{1}{2}$.

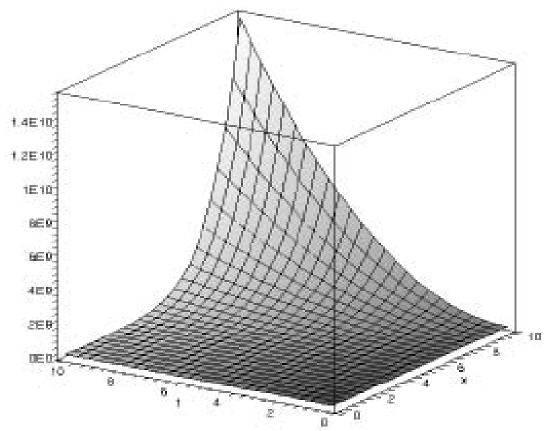

Figure 4: Solution of Eq.(29)obtained by ADM method for $\beta=1$.

4 Conclusion

In this paper, the ADM has been successfully applied to derive explicit numerical solutions for the time-and space-fractional Cahn-Hilliard equation. The above procedure shows that the ADM method is efficient and powerful in solving wide classes of equations in particular evolution fractional order equations.

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