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FACULTY OF EXACTS CIENCES AND COMPUTER SCIENCE

Department of Mathematics and Computer Science

Course handout

Presented by:

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TITLE

Introduction to the Didactics of Mathematics

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1 Introduction

— Definition, scope, and objectives.
— Didactics and human sciences :

1. Didactics and pedagogy.
2. Didactics and psychology.
3. Didactics and social psychology.
4. Didactics and epistemology.

2 Key Concepts

— The didactic triangle.
— Didactic transposition.
— Students' conceptions and representations.
— Didactic obstacle and objective-obstacle.
— The didactic contract.
— The didactic sequence / example of a problem-based situation.

3 Research Methodology in Didactics

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8 mai 2025

5 Preface

This book has been designed with a specific purpose : to serve as a practical and straightforward guide, a collection of solved exercises with course summaries, intended for third-year undergraduate mathematics students studying the Didactics module. It contains the essential concepts of the course, illustrated with examples, while adhering to the curriculum set by the ministry.

It is by no means a substitute for a complete theoretical course ; rather, it is meant to accompany it progressively to ensure a better understanding and assimilation of the material. A variety of application exercises of different difficulty levels are provided at the end of each chapter to clarify the reasoning involved, allowing students to test their knowledge and prepare for exams and final assessments.

The result of many years of teaching, this work, while remaining modest, is divided into four chapters, with content and order following the framework established by the ministry.

It also stems from the observation of the difficulties students face when approaching mathematics at the undergraduate level, which can be summarized as follows :

— Difficulty in understanding a given problem statement ;
— Difficulty in formulating and structuring logical reasoning ;
— Difficulty in applying a methodical approach and identifying underlying ideas.

Introduction : Course Handout on Didactics (II)

The **Didactics Module (2)** is at the heart of the training program, which makes it crucial to carefully select the content and activities.

If didactics is seen as a discipline aimed at acquiring knowledge and the content of the subject being taught, it also plays a key role in training by guiding both **know-how** and **interpersonal skills**, which must be adapted based on the desired learning outcomes and students' future professional contexts.

Students are expected to :

— Understand the fundamental concepts of didactics,
— Grasp the evolution of methodologies and approaches in language teaching,
— Familiarize themselves with teaching practices,
— Comprehend the relationship between knowledge and learning,
— Reflect on the content to be taught, as well as the forms and functions of language practices in knowledge construction.

Ultimately, the objective is to guide them toward **developing a didactic approach** in their work.

Targeted Competencies :

— Develop knowledge of didactic terminology and teaching practices.
— Analyze and theorize teaching and learning phenomena related to specific disciplinary dimensions (French).
— Differentiate between disciplinary approaches in education sciences (sociological, didactic, psychological, etc.) and understand the contributions and limitations of each in analyzing educational and social phenomena.
— Develop analytical and synthesis skills necessary for reading **scientific texts on didactics**.
— Critically examine educational frameworks, situations, discourse, and attitudes in the world of education.
— Identify and observe activities and training methods in school settings to **better understand educational practices**.

6 Introduction

The didactics of a discipline is the science that studies, for a particular field, the phenomena of teaching, the conditions for the transmission of culture specific to an institution, and the conditions for acquiring knowledge by a learner.

6.1 Definition, Scope, and Objects

The adjective *didactic* has long been used to characterize works aimed at instruction.

Today, nearly 350 years after the publication of *The Great Didactic* by Comenius (1657), the noun *didactics* has two meanings :

1. In its common usage, the expression "language didactics," "mathematics didactics," "mechanics didactics," etc., refers to the use of teaching techniques and methods specific to each discipline.

   The techniques used naturally vary depending on the subject, as they directly depend on the content to be taught.

   — Language teaching emphasizes audio-oral techniques.
   — Physical sciences teaching follows an experimental approach.
   — Economics teaching relies on case studies.

   The chosen pedagogical techniques, their adaptation to the characteristics of the discipline being taught, and their structure form the didactics of the discipline.
2. In its modern sense, didactics studies the interactions that may occur in a teaching/learning situation between :
   — A well-defined body of knowledge,
   — A teacher delivering this knowledge, and
   — A student receiving it.

   Didactics no longer simply follows pre-established teaching models ; it requires :
   — A teacher's epistemological reflection on the nature of the knowledge they will teach, and
   — Consideration of the learner's perceptions of this knowledge (*learner's epistemology*).

As we can see, the meaning of the word *didactics* has evolved significantly over the past fifteen years.

Two concurrent factors seem to explain the growing interest in this concept today :

6.2 Research Findings

No more than thirty years ago (between 1975-1980), researchers became aware of a phenomenon that had previously gone unnoticed. It was discovered that :

> "The percentage of French adults who had completed regular schooling but still exhibited severe gaps in basic knowledge was far from negligible." (1)

Giordan and De Vecchi (2) present the results of a study conducted by Kapferer and Dubois (3) :

> "Participants were asked whether they believed the statement 'The Sun revolves around the Earth' was true or false. The results were almost unbelievable."

Response	Percentage
Completely true	31.5%
Probably true	8.9%
Probably false	5.2%
Completely false	52.1%
Doesn't know	5.4%

TABLE 1 – Survey results on the statement "The Sun revolves around the Earth"

Based on these percentages, it is estimated that more than fourteen million adults believe this statement to be true. These results are surprising. Even more astonishing is the fact that a significant percentage of individuals who agreed with the statement had completed general or technical secondary education.

Level of Education	Percentage Believing the Statement
Primary	52.4%
Upper Primary	41.8%
Technical / Commercial	30.7%
Secondary	37.5%
Higher Education	14.6%

TABLE 2 – Belief in the statement "The Sun revolves around the Earth" by education level

Shocked by these findings, researchers continued their investigation and discovered that many basic concepts in disciplines studied by students were not actually well understood. More than a quarter of physics students believed that when walking across a room while holding a golf ball, one should drop the ball directly above the target rather than slightly ahead if they wish to hit it. (4)

It thus appears that, in many cases,

> "The knowledge memorized at school or outside of school coexists with a persistent prior understanding, which at best is slightly altered to accommodate new learning but remains largely intact." (5)

As a result, new knowledge is often not fully integrated, and pre-existing conceptions resist change.

Researchers and educators across various disciplines have therefore begun working to identify learners' preconceptions that hinder the acquisition of new knowledge. The current focus is on analyzing the nature of these obstacles, revisiting the epistemology of each discipline, and tracing the transformation of scientific knowledge as it undergoes didactic transposition and subsequent teaching—perhaps losing some of its original identity along the way.

Thus, modern disciplinary didactics refers to a broad spectrum of research that simultaneously challenges :

— Researchers, concerning the epistemological obstacles in their discipline.
— Educators, regarding their necessarily reductive teaching choices.
— Learners, in relation to their understanding of knowledge.

6.3 b. Socio-Institutional Context

The early 1980s, undeniably influenced by the discourse of generalist educators and education theorists, saw the emergence of yet another set of pedagogical terminology—one that secondary school teachers, primarily specialists in their respective disciplines, were reluctant to accept.

This discourse primarily revolved around concepts such as mastery-based learning objectives, differentiated pedagogy, formative assessment, and remediation. However, except in a few cases, these innovations failed to significantly influence the teaching profession.

7 Didactics and Pedagogy

With the work of didacticians, content regains its prominence, allowing for what can finally be considered "true pedagogy," based on "serious" research—conducted by peers on a well-known and well-mastered discipline.

Opposing pedagogy and didactics is absurd ; these two fields are obviously complementary. Any practitioner seeking to enhance the effectiveness of their teaching would benefit from engaging with the findings published by both branches of research.

The shift initiated by modern didactics, influenced by the renewal of psychology—now focusing less on behavior and more on cognition (information processing, problem-solving, representation, intentionality, goals, contextual effects...)—has some parallels with the movement undertaken a century ago by Progressive Education : placing the student at the heart of the educational process.

The student of the 1920s, characterized by "activity," play, needs, motivations, interests, and projects, has since evolved. Today, the student is also understood in terms of cognitive schemas, representations, obstacles, sociocognitive conflicts, relationship to knowledge, and pseudo-concepts. In essence, the student has become a cognitive learner.

8 Types of Didactics

Alongside researcher-didacticians, there are other actors who rightfully claim a role in didactics : specialized teachers or trainers, inspectors, and innovators.

Thus, we can distinguish at least three types of didactics :

— **Practical Didactics**, focused on classroom application.
— **Normative Didactics**, which establishes teaching standards and guidelines.
— **Critical and Prospective Didactics**, developed by innovators and researchers, which challenges and anticipates future educational needs.

Each of these approaches must address its challenges within a specific context.

However, a fundamental common viewpoint emerges : didactics is always concerned with teaching and learning while fully considering the specificity of content. In this sense, every didactician bears a "responsibility" toward the content—whether in its creation, adaptation, legitimacy, or integrity.

9 Didactics in the Field of School Disciplines

Didactics usually operate within the scope of school disciplines ; hence, we often speak of discipline-specific didactics.

In formal education, disciplines possess a certain degree of permanence and unity that grants them a recognized social existence. It would be somewhat futile to develop a didactics of probability independent of other branches of mathematics or a didactics of volleyball independent of sports education.

At the other extreme, the question of general didactics arises. This is a significant question for elementary education and teacher training. Research must acknowledge that, so far, no general didactics exist, likely due to a lack of a relevant research object. However, comparative didactics research is already proving fruitful, starting from the hypothesis of content-based differences and highlighting the divergences and convergences between disciplines, levels, education systems, and countries.

Martinand J.-L. in Champy, Ph. & Etévé, Ch., Encyclopedic Dictionary of Education and Training, Paris, Nathan, p.255.

10 Didactics and Human Sciences

Going from the narrowest to the broadest definition, didactics can be understood as follows :

1. A reflection on the objects of teaching. It focuses on their cognitive nature : knowledge or skills ; their epistemological status : scholarly knowledge or social knowledge ; their construction methodology : transposition or knowledge development ; and their institutional history. The dominant perspective here is epistemological.
2. Research on the conditions for knowledge appropriation. It is less concerned with concepts and notions themselves than with their construction in learning, the prerequisites they require, learners' common representations, and the various learning obstacles they may present. The dominant perspective here is psychological.
3. Research on didactic intervention. Didactics link the previous points to the teacher's tasks, the organization of teaching situations, the construction of didactic cycles or sequences, adaptation to the audience type—in short, an approach to the classroom and its specific functioning. The dominant perspective here is psychosociological.

11 Didactics and Pedagogy

Pedagogy is defined as any activity undertaken by an individual to develop specific learning in others.

To clarify the meaning of the term pedagogy, it is essential to differentiate it from educational sciences and didactics.

The pedagogue seeks to answer questions directly related to their educational practice : "What do we know about human learning that allows us to construct effective teaching strategies ?" or "What would be the most effective teaching method for a particular type of learning ?" or even "How can we promote learning to read through a small classroom newspaper ?" The pedagogue appears as a practitioner primarily concerned with the effectiveness of their action. They are a field professional who continuously solves concrete teaching and learning problems. The primary source of their "pedagogical intuition" remains action and experimentation, from which they derive validation and encouragement.

The researcher in educational sciences, on the other hand, addresses a different set of questions, likely less linked to teaching practice but crucial for the pedagogue : "What are the causes of school failure ?" "What is the relationship between learning and information processing theories ?" or "To learn, must the learner make mistakes ?" Their primary concern is improving our understanding of phenomena that more or less directly influence educational action (see Educational Sciences).

The didactician, meanwhile, is above all a specialist in teaching their discipline. They primarily examine the notions, concepts, and principles within their discipline that must be transformed into teachable content. They also assess the level of their students (individual difficulties, personal representations, etc.) to identify epistemological or psychological obstacles that need to be overcome to facilitate learning. The work of the didactician is thus essentially a matter of information processing : identifying and transforming "scholarly knowledge" (reference knowledge) into "knowledge to be taught" (see Didactic Transposition).

Clearly, these three approaches to pedagogical action are not mutually exclusive. For instance, an action-research situation integrates these three dimensions.

In its contemporary framework, pedagogical action calls for a highly complex synergy of knowledge and skills from various fields of experience. Pedagogues, didacticians, and educational science researchers are increasingly encouraged to exchange information while maintaining shared objectives : how to truly facilitate learning, develop individuals, reduce school failure ? How to promote knowledge transfer ? How to implement more efficient learning regulation systems ? Or even, how to transform a Gaussian curve into a J-curve, or quite simply, how to make children love school ?

A difficulty remains, however : while the objectives and goals of pedagogical action can be shared by most stakeholders, the strategies to achieve them vary considerably and are sometimes even opposed.

12 What is Pedagogy ?

Every human behavior, every action, is always subordinated to a set of beliefs. Pedagogy is no exception to this rule. It is a complex action, guided by values (the educator's view of humanity, society, and their mutual relationship) and by hypotheses concerning the development of individuals, their way of constructing themselves, and projecting themselves into a harmonious social life.

Since these approaches are naturally infused with ideology, educators have devised very different systems based on their personal conceptions, objectives, and the audience they address. This has led to the development of various types of pedagogy, such as :

— Active pedagogy
— Adult pedagogy
— Contract-based pedagogy
— Project-based pedagogy
— Discovery-based pedagogy
— Differentiated pedagogy

13 Bruner's Approach

Bruner reminds us of some essential reference points for "working in pedagogy." These reference points are structured in a way similar to cybernetic programming : action, analysis of action, regulatory feedback, and correction of action.

For practitioners like us, Bruner's language, tone, and the generosity that shines through his text remain a valuable guide :

> "Allow me to take a very pragmatic position. Develop the best pedagogy possible. See what results you obtain. Then analyze the nature of what has been successful in what you have done. [...] The best thing you can do at any given moment, in my opinion, is to create a pedagogical 'treatment' that works very well and then reflect on it, formulating hypotheses to determine what you have done. [...] With a bit of psychology, a bit of common sense, and a bit of luck, you can achieve a learning outcome that deserves to be studied. Then you will simplify and experiment. But first, invent and observe."

Source : Raynal Fr., Rieunier A., *Pedagogy : Dictionary of Key Concepts*, Paris, ESF publisher, 1998, pp. 263-265.

14 Definition of Pedagogy and Didactics

> "In the school environment, 'pedagogy' refers to everything related to the art of conducting and managing a classroom, encompassing what was once called discipline, but also the organization and significance of work. It involves both the practice of this art and the reflection on its resources and purposes. Didactics, on the other hand, concerns the art or manner of teaching the specific concepts of each subject, including some difficulties inherent to a particular domain within a discipline."

Source : Cornu L., Vergnioux A., *Didactics in Question*, Paris, Hachette Education / CNDP, 1992, pp. 9-10.

15 Didactic Approaches

> "Didactic approaches operate, on one hand, upstream of pedagogical reflection by considering teaching content as an object of study. Didactics then enable the identification of the key concepts that function within a discipline and the analysis of their relationships. It examines their history, their respective refinements, and the modalities of their introduction into teaching. It also studies the social functioning of these concepts and the social practices to which they refer... Concepts such as conceptual frameworks, levels of formulation, didactic transposition, and reference social practices are all present here. On the other hand, didactics also work downstream by deepening the analysis of classroom situations."

Source : Bruner J., "Some Elements of Discovery" (1966) in Shulman L.S. and Keislar E.R., *The Discovery-Based Pedagogy*, Paris, ESF publisher, 1973, pp. 99-100.

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Didactics and Psychology

16 Understanding the Learning Process

To better understand from within how the learning process works and what is at stake, the study of students' representations, their reasoning methods, and how they interpret the teacher's expectations is crucial. Additionally, analyzing the teacher's mode of intervention allows for the suggestion of a range of possible interventions rather than confining them to a single mode of action.
Source : Astolfi J-P., Develay M., *La didactique des sciences*, Paris, PUF, 1986, "Que sais-je ?", No. 2448, pp. 9-10.

17 Didactics and Psychology

> "Didactics is defined primarily by research on the conditions of knowledge acquisition. It focuses less on concepts and notions themselves than on their construction, the prerequisites they require, the common representations held by learners, and the different types of obstacles to learning they may present... The dominant perspective is psychological." (Halté, 1992)

17.1 The Origins of Psychology

The term *psychology*, coined by the German scholar Melanchthon in the 15th century from the Greek *psukhê* (soul) and *logos* (science), referred until the early 20th century to the study of the soul and mental activity.

Since Plato and Aristotle, psychology has always been intertwined with philosophy, and it was not until the 19th century that it became an autonomous science. Psychology then evolved into the scientific study of behavior and mental processes, seeking to answer the fundamental question :

"Why does man act as he does ?"

To answer this question, several perspectives have emerged :

17.2 Neurobiological Perspective

This approach attempts to identify the brain components responsible for specific behaviors when stimulated electrically. It aims to create a cerebral localization map and an electrochemical theory of behavior (see Neurosciences).

17.3 Behaviorist Perspective

Behaviorists focus solely on observable aspects of behavior. They study the stimuli presented by the environment and the responses produced by individuals. They consider reinforcement (reward or punishment) to be one of the key elements in behavior control.

For them, the fundamental behavior model is :

Stimulus - Response - Reinforcement

17.4 Cognitivist Perspective

Cognitivists view humans as vast information-processing systems, where behavior is primarily explained by perception and the processing of incoming information. They focus on what happens between the stimulus and the response : motivation, perception, imagery, storage and retrieval of information in long-term memory, activity control, and most importantly, *expectations*, a key concept in cognitivism (see Information Processing).

17.5 Psychoanalytic Perspective

For Freud and psychoanalysts, behavior is primarily explained through unconscious conflicting processes that gradually develop during early childhood.

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Source : Astolfi J-P., Develay M., *La didactique des sciences*, Paris, PUF, 1986, "Que sais-je ?", No. 2448, pp. 9-10.

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19.5 Psychoanalytic Perspective

For Freud and psychoanalysts, behavior is primarily explained through unconscious conflicting processes that gradually develop during early childhood, during the awakening of sexuality, in parent-child relationships, or through interactions with the external world (see Freud, Psychoanalysis, Clinical Psychology).

19.6 Phenomenological or Humanistic Perspective

According to these psychologists, the best way to explain an individual's behavior is through direct conversation. By conducting a non-directive clinical interview, they aim to understand the individual's motivations based on how they perceive phenomena.

20 The Evolution of Psychological Paradigms

Psychology thus brings together multiple perspectives, all striving—despite significant methodological and epistemological differences—to understand, describe, and explain *human behavior*. These fundamentally different approaches have coexisted for nearly a century, sometimes ignoring each other, sometimes fiercely opposing each other.

While the behaviorist paradigm dominated psychology for almost a quarter of a century—introducing a research methodology previously unknown in the humanities—some theoretical movements resisted the "stimulus-response" explanation : Freud's psychoanalysis, Janet's clinical psychology, the structuralism of Gestalt psychologists and Piaget (Gestaltism and Constructivism), and the developmental psychology of Wallon, Vygotsky, and Piaget.

Since the 1960s, on the ashes of behaviorism, **cognitivism** has emerged as the dominant paradigm. By introducing the concepts of symbolic representation and processing of these representations, it reconnected with the study of mental activities but on radically different foundations.

> "Cognitive psychology is the science of information processing in humans. Specialists in this field study the types of information we store in memory, the processes involved in acquiring, transforming, and using it. This is a vast field of study that extends from perception to problem-solving, including memory and learning. This research movement grew significantly after World War II, following the work of Shannon and Weaver (1949) on information theory." (1)

21 Cognitive Conflict and Learning

The experimental setup typically involves :
— A pre-test to verify that the child or children participating in the experiment cannot solve the problem on their own.
— An interaction phase between subjects of the same or different developmental levels.
— A post-test to assess any progress made.

21.1 Interaction Between Two Children

— If both children are at the same developmental stage, researchers observe their interactions.
— If one child is slightly more advanced than the other, the interaction is studied in detail.

Since each child proposes different solutions, a **social conflict** arises. This **socio-cognitive conflict** is of great interest to researchers because it is believed to be a key driver of cognitive progress.

In this situation, the child in conflict with their peer is compelled to reconsider their point of view in order to understand their companion's perspective. This process of **decentration, reconstruction, and synthesis of viewpoints** is thought to facilitate problem-solving and intellectual development.

However, this effect is only observed if the child is in a **sensitive stage**, meaning they are in a latent phase preceding their transition to the next developmental stage.

21.2 Interaction Between a Child and an Adult

In this case, the adult plays a guiding role, similar to Piagetian experiments, offering suggestions or counterproposals. Researchers then analyze the influence of social status on decision-making and the causes of any observed development.
Source : Raynal Fr., Rieunier A., *Pédagogie : dictionnaire des concepts clés*, Paris, ESF éditeur, 1998, pp. 308-309.

22 Didactics and Epistemology

> "Didactics is defined notably through reflection on the objects of teaching. It examines their cognitive nature : knowledge or know-how... ; their epistemolo-

gical status : scholarly knowledge or social knowledge... ; the methodology of their construction : transposition or elaboration of knowledge... ; their institutional history... The dominant perspective here is epistemological." (Halté, 1992)

22.1 What is Epistemology ?

Epistemology is the study of knowledge. However, it is a complex concept, and many students struggle to grasp it upon their first encounter.

The study of knowledge can be approached from two main perspectives :

The first approach consists of epistemologists who attempt to answer the question :

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23 Epistemology and Knowledge Development

This is the case with Piaget, and the answers to this question are of primary interest to educators.

> "The central problem of epistemology is to determine whether knowledge is merely a pure recording by the subject of data that is already fully organized independently of them in the external world (physical or ideal), or whether the subject actively intervenes in knowledge and the organization of objects, as Kant believed." [**?**]

24 Two Approaches to Epistemology

24.1 First Approach

The first approach concerns epistemologists who focus on this fundamental question about the nature of knowledge.

24.2 Second Approach

The second group consists of epistemologists who seek to answer the question :

> "How do knowledge and understanding develop in a particular field or across different domains ?"

This is the perspective of Bachelard and Karl Popper. In this context, epistemology is synonymous with the philosophy of science.

> "Epistemology, a late branch of philosophy, aims to study the hypotheses and methods of various scientific disciplines in order to define their underlying logic and determine the objective scope of their results. It therefore relies both on the history of science and on logic." [**?**]

25 Key Concepts

Like any other field of knowledge, didactics has developed concepts to designate the objects it studies.

A **concept** is a general and abstract idea attributed to a category of objects sharing common characteristics, allowing for the organization of knowledge.

Our environment consists of *"objects,"* which can be either :

— **Concrete** (humans, trees, cars, houses...)

— **Abstract** (freedom, resilience, sovereignty, potential difference...)

To understand and represent this world—and therefore be able to act upon it in thought—humans create **concepts**. This enables them to formulate hypotheses for action, assess their consequences, adjust these hypotheses, imagine possibilities beyond reality, and weigh pros and cons. Often, this process is far less costly than real action.

Concepts are commonly divided into two categories :

1. **Concrete concepts** : These are concepts whose instances can be touched, manipulated, or shown.
   — Examples : dog, car, shoes, ammeter, flower, human, etc.
2. **Abstract concepts** : These refer to ideas or constructs that cannot be physically manipulated.

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1. **Concrete concepts** : These are concepts whose instances can be touched, manipulated, or shown.
   — Examples : dog, car, shoes, ammeter, flower, human, etc.
   Learning a concrete concept means being able to classify a previously unseen example within the appropriate category. If a European child sees a frangipani flower for the first time and says "this is a flower," they demonstrate that they have learned the concept of a flower.

   In a school setting, verifying that a concept has been learned can be done by asking a student to classify an unseen representative of that concept into its category. If this is done correctly multiple times, one can infer that the concept has been learned. Another way is to ask the student to provide a new example of the concept.
2. **Abstract or defined concepts** : According to Gagné, a defined concept "is a rule that allows objects or events to be classified."[**?**]

   Examples :
   — "A nephew is the son of a brother or sister."
   — "Independence is the state of a person or community that is not subject to any authority."
   Learning an abstract concept means being able to classify an unseen representative of this concept into the appropriate category or to classify it as an exception.

   Example : "Given any number, determine whether it is a prime number or not."

27 The Didactic Triangle

Didactics considers all the participants in the didactic relationship, a specific interaction between a teacher, a student, and knowledge within a given educational environment

and time frame.

Didactic Triangle

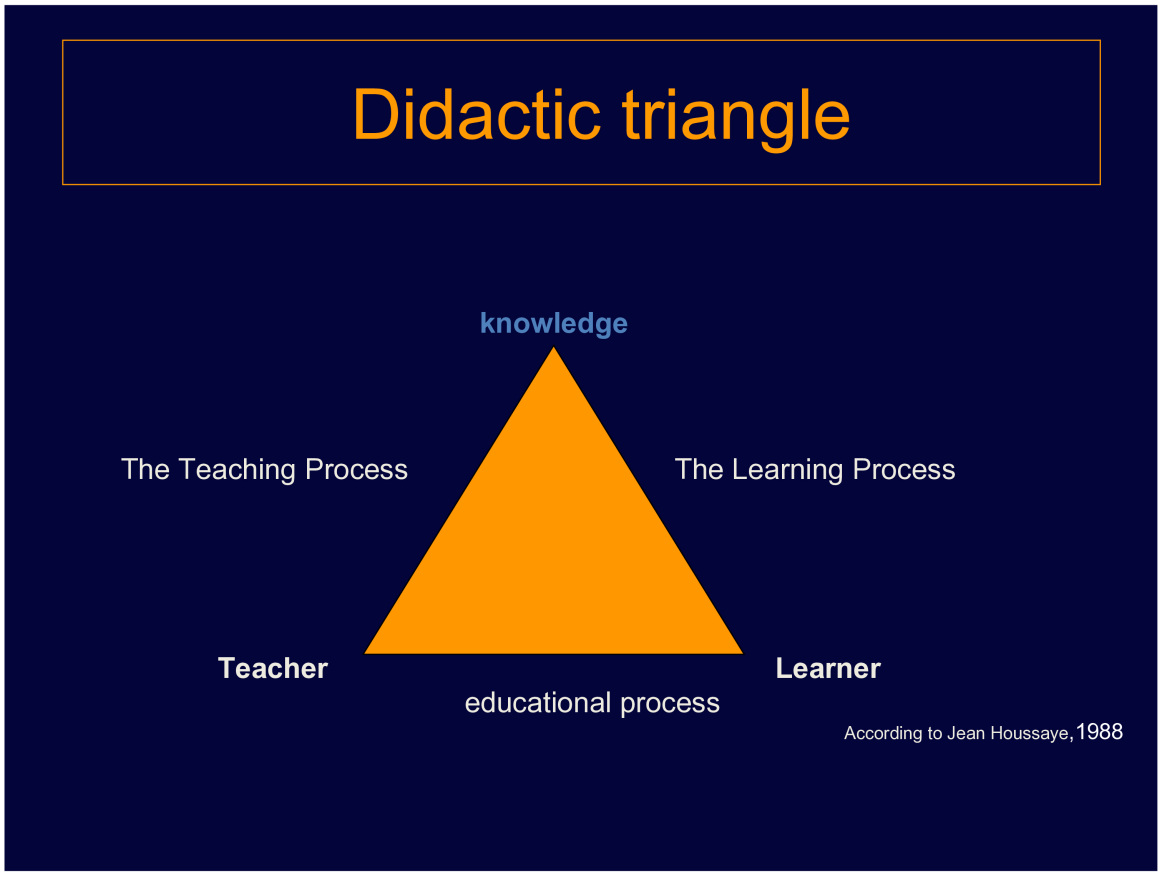


FIGURE 1 – Description de l'image

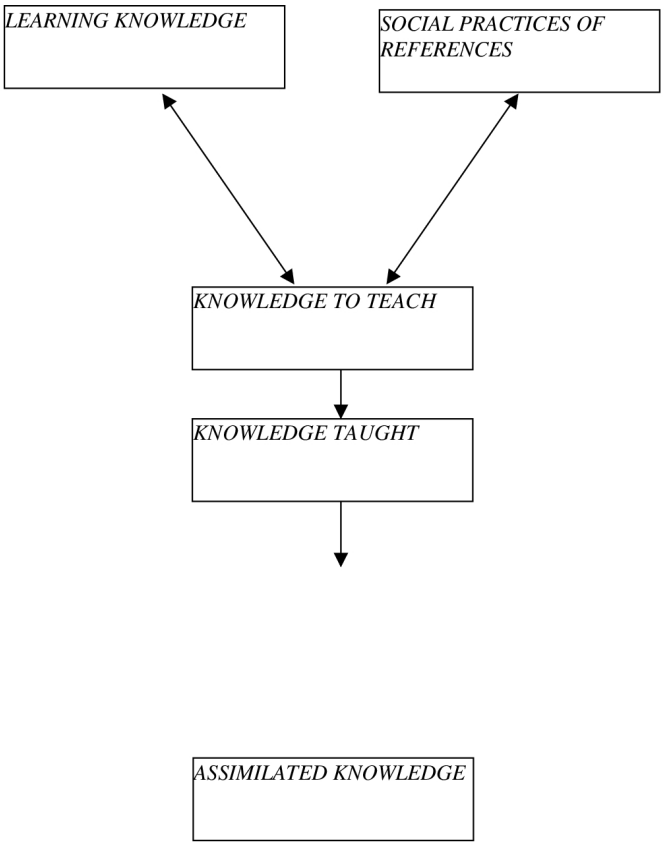
The didactic triangle seeks to clarify the object and specificity of didactics. It represents the relationships between the teacher, the student, and knowledge.

28 Didactic Transposition

Designed to apprehend and model a complex situation. Of course, such "modeling" is not immune to criticism. One can refer to the critiques by Laurence Cornu and Alain Vergnioux in *La didactique en questions ?*, Paris, Hachette Education, 1992, page 120 and following.

Teaching is the result of a didactic process governed by specific constraints : a distinction is made between scholarly knowledge (as it emerges from research) and taught knowledge (the one observed in classroom practices).

Didactic transposition consists of the "general mechanisms that allow the transition from an object of knowledge to an object of teaching."



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29 Knowledge Hierarchy

Borrowed from sociologist Michel Verret (1), the concept of didactic transposition was introduced into didactics with great success. (2) It is becoming increasingly clear that what is taught is not merely a simplified replica of scholarly knowledge, but rather a specific reconstruction for the school system. This reconstruction, along with its stages

and processes, is what we call didactic transposition.

This concept aligns with what Halbwachs had already pointed out in a classic article aptly titled : *"The Physics of the Teacher, Between the Physics of the Physicist and the Physics of the Student"* (3). This pioneer understood that the profession of a physics teacher cannot be reduced to the practice of a physicist who simply teaches !

Initial Conceptions of Students and Didactic Obstacle

The research on initial conceptions of students and their resistance to teaching shows that a true learning process is defined as much by the conceptual transformations it brings about in the individual as by the body of knowledge provided. (J.-P. Astolfi, 1989).

Teaching from Conceptions :

Numerous empirical studies conducted in the 1980s (Ph. Jonnaert, G. DeVecchi, etc.) demonstrate the necessity of taking students' conceptions into account in the teaching-learning process.

In a thesis defended in 1986, Ph. Jonnaert experimentally showed the didactic effectiveness of taking conceptions into account by comparing the performance of two groups of students in mathematics.

Gérard De Vecchi (1984) reached the same results in his experiments on the biological concept of digestion.

Taking students' conceptions into account is generally done "by activating socio-cognitive conflicts within the class through problem-solving situations" (M. Develay, 1989).

Conceptual Change :

How can learners transition from pre-scientific conceptions to scientific conceptions ? How can we help them make conceptual changes ?

Three schools of thought currently dominate the research on conceptual change :

- **The Bachelardian school**, for whom conceptual change occurs through a break with preconceptions. J.-P. Astolfi represents this trend in the didactics of science.
- **The American school** : Based on the theory of ontological categories, Chi and Slotta (1993) developed a theory of conceptual change according to which the resistance of preconceptions to teaching arises because the two competing contents do not belong to the same category.
- **The Theory of Contextual Change** : This theory posits that conceptual change becomes possible when learners are told that each context (school context/social context) corresponds to a different way of framing problems (K. Bouraoui, 1998 ; Cheikhrouhou, 1997).

Example :

Description of a classroom situation : In a first-grade class, each student placed a label with their name on the desk. The teacher mixed these labels and asked the students

to check if there were as many boys as girls in the math workshop that morning. The students sorted the labels into two piles : "girls" and "boys," counting the number of girls and boys present. The teacher also placed a label with her name on the desk. It was observed that some students did not count the teacher among the girls present in the class or, conversely, gave her a value higher than "one" when counting. Below are the transcriptions of conversations with two of these students.

Excerpts from exchanges between two students and the observer :

The observer asked Nicolas why he was not considering the teacher's label.
— **Obs.** : Why don't you count Monique (the teacher) ?
— **N.** : Because it's not fair to the boys.
— **Obs.** : What do you mean ?
— **N.** : She's worth more than a student, so it's not fair.
— **Obs.** : What do you mean by "she's worth more" ?

Continuation of the Classroom Situation :

— **N.** : She's not just worth one like us, she's worth more.
— **Obs.** : So, how much do you think she's worth ?
— **M.** : I don't know, but it must be a lot because she's the teacher.
— **F.** : I counted 12 boys and 12 girls. It's the same for both.
— **M.** : You're cheating because there are more boys.
— (N. lines up 12 names of boys and 10 names of girls)
— **Obs.** : So, Nicolas, how many boys and how many girls are there ?
— **N.** : 12 boys and 10 girls.
— **F.** : You have to count Monique too, she's a girl like us.
— **M.** : No, she's the teacher, it's not fair !
— **Obs.** : What's not fair ?
— **M.** : A teacher is not like us, so we don't count her.
— **Obs.** : Why don't you want to count her ?
— **M.** : Because then there would be more girls than boys, and that's not true... We're the majority in the class, not the girls.
— **Obs.** : Fabienne, can you count out loud the number of names you have in front of you ?
— **F.** : Yes.
— (F. counts the names aloud)
— **F.** : There are 12 cards with boys' names and 11 cards with girls' names.
— **Obs.** : Why are you saying there are 12 boys and 12 girls instead of 12 boys and 11 girls ?
— (Silence, F. recounts and thinks, with Nicolas looking upset)
— **F.** : Because Monique is worth 2. Because she's the teacher.
— **N.** : That's not fair, we can't count Monique.
— (Excerpt from an observation made in a first-grade class in the Province of Hainaut, French-speaking Belgium ; this is the transcription of a videotaped sequence.)

Nicolas and Fabienne are confronted with an identical situation :

A set of 23 labels, 12 representing the boys' names in the class and 11 representing the girls' names, including that of the teacher.

Theoretically, both should come up with the same representation of this situation... but they don't !

While Nicolas counts a total of 22 labels (12 boys and 10 girls), Fabienne counts 24 (12 boys and 12 girls), whereas the observer sees 23 (12 boys and 11 girls). Who is right ?

Each organizes this situation based on their personal experiences, their lived experiences, and the meaning they wish to attribute to it. Ultimately, Nicolas constructs his situation, Fabienne constructs hers, and the observer constructs the situation that interests them. This is the first dimension of the constructivist paradigm : each person is constantly constructing their own reality.

The dialogue that unfolds between Nicolas and Fabienne shows that their representations diverge and rest on different logics. For Nicolas, the label representing the teacher's name cannot be considered (it therefore counts as 0), whereas for Fabienne, conversely, this label is worth twice the value of any other label. We can imagine other debates, other rivalries behind these comments.

Let us retain for now that these two first-grade students have developed different representations of the same reality. They even arrive at divergent counts of the objects they are confronted with.

This situation is not without recalling the experiment described by Kuhn (1983). We summarize it in the lines that follow. Wanting to know what atomic theory represents for scientists, a North American researcher asks two specialists recognized by the international scientific community in their respective fields : chemistry and physics. He asks them if a helium atom is or is not a...

Helium Atom and Molecular Theory :

The answer from each expert does not align. For the chemist, a helium atom is a molecule, and they justify this answer by referring to the kinetic theory of gases. For the physicist, a helium atom is not a molecule. They argue that they cannot observe its molecular spectrum.

Do these answers contradict each other ? Are we at an impasse ? No, certainly not. This situation is quite common. Both the chemist and the physicist construct their definition of a helium atom based on their reference field.

We experience similar situations every day ! Consider a trivial situation : If a geographer, a watercolorist, a geologist, an urban planner, a photographer, a farmer, an ecologist, a lumberjack, and we ourselves—looking for a quiet spot to picnic with our family—are stopped in front of the same countryside site, we will certainly not look at it the same way. No doubt, while the photographer will look for light, the lumberjack will look at the trees. Where we search for a shaded spot for lunch, the watercolorist will look for colors. Where the farmer finds fertile soil to clear, the ecologist will identify plant species to protect...

So, what happens in this case ? Is each person constructing their own vision of ontological reality ? And is it still relevant to talk about ontological reality in this case ?

Constructivists' Perspective :

It is from the constructivists that we find an initial answer to this question. Constructivists abandon the idea of knowledge as an exact copy of the external reality, the ontological reality. For them, it is normal that the representations Nicolas and Fabienne have of the situation they are confronted with are not an exact reproduction of it. Constantly, each individual seeks to construct the world by constructing themselves in such a way that they can fit into it and adopt a viable position (Pépin, 1994). From this perspective, knowledge is constructed by the subject themselves through the experiences they live in their environment, but also from those they have already lived :

> "(...)" the notion of knowledge-reflection or knowledge-reading, which carries a sense of imprint or image of the real in our knowledge, seems quite illusory, even utopian. Knowledge, understood as a more or less complete copy of reality, no longer makes sense because we only have access to reality through our representations. (Ruel, 1994)

Ontological Reality Cannot Be Copied !

Thus, ontological reality cannot be copied ! The knowledge of the learning subject is not like a photo album, a series of snapshots that are exact copies of the external world. It's impossible ! The learning subject does not photograph the world ; they continuously reconstruct it while reconstructing themselves. From this perspective, knowledge is not passively transmissible ; it is constructed by the subject who learns. This constructivist postulate has a very significant impact on our reflections about teaching and learning.

Didactic Obstacle and Objective-Obstacle

This concept was introduced in pedagogy by J-L Martinand to renew both the concept of the pedagogical objective and the methodology for creating lessons.

Martinand's Critique of Atomistic Objectives

Martinand, a researcher in the didactics of science, is not satisfied with the atomistic drift provoked by Mager's proposal (see Mager, Objective), and regrets the fragmentation of intermediate and specific (low-level) objectives into a multitude of micro-objectives that lack significant value for learning. He argues that "objectives must be limited in number if they are to be genuinely useful for teachers. Behavioral-type objectives are always far too numerous, momentary, and scattered to be a permanent aid." (1)

By analyzing the content to be taught, an epistemological reflection on the concepts and principles of the discipline is provoked : "We can identify a number of decisive advances, not spontaneously acquired, but which have significance from the perspective of scientific or technological thinking." (2) Therefore, objectives must be defined in relation to the difficulties of modifying representations, which themselves are linked to epistemological obstacles as defined by Bachelard (Bachelard, 1938).

Martinand's approach, adopted by many authors and researchers in pedagogy, originates from the renewal of the didactics of science observed since the 1980s. It reconnects with the taxonomic approach, which analyzes the type of intellectual activity the teacher seeks to develop when designing learning situations.

Methodology for Constructing a Learning Situation

Astolfi and Develay propose the following methodology for constructing a learning situation based on the concept of obstacles :

1. Identify the "obstacles."
2. Define the intellectual progress.
3. Select the "surmountable" obstacle based on the students' level.
4. Define the objective from this obstacle.
5. Place the objective within a taxonomy.
6. Translate this objective into operational terms.
7. Design the strategy.
8. Design remediation. (3)

Thus, the objective-obstacles enable :

— For the student : to better understand the "cognitive leap" expected of them.

— For the trainer : to reformulate their objectives based on the core concepts of the discipline, which represent a major difficulty in conceptualization for students.

In practice, objective-obstacles introduce greater flexibility in organizing learning situations. "Compartmentalization" is reorganized around problem-solving situations or core objectives, whose treatment requires a redistribution of learning activities into longer and more differentiated time segments.

Objective-obstacles are characteristic of a reorientation of action at the level of intermediate objectives ; far from symbolizing a return to content, they directly tackle the central problem in the psychology of learning, viewed through a cognitive approach—namely, the genesis of core concepts and the construction of representations within a given discipline.

The Didactic Contract

The didactic contract is an implicit agreement between the teacher and the students that guarantees, as long as both parties respect the clauses of the contract, that exchanges in the classroom will occur without major difficulties. This implicit contract legitimizes the statuses, roles, and role expectations of each person toward the other, provided there is no "deception" or "misinterpretation."

The concept of the didactic contract is very different from that of the pedagogical contract, which explicitly aims to clarify the objectives and school requirements for the students. On the contrary, the term "didactic contract," originally introduced by Brousseau in the didactics of mathematics, explains that there is necessarily some opacity in the "didactic system" connecting knowledge, the student, and the teacher. In reference to Jean-Jacques Rousseau's social contract, the idea is that the didactic contract preexists the teaching situation and, in a sense, overdetermines it. This is not because the teacher seeks to hide something from the students, but because both the teacher and the students are caught in this contract that binds them. The foundation of the didactic contract does not concern the nature of the pedagogical relationship : it is first and foremost epistemological. It notably arises from the obstacles students face while learning, and from the didactic situation itself. This situation must indeed confront students with obstacles, without the teacher substituting them to overcome those obstacles. If the teacher too explicitly defines what is expected from the students, the intellectual task they are assigned

collapses, and they are left to mechanically perform a sequence of operations. This is what Brousseau calls the "Topaze effect," referring to Pagnol's character, who makes students do a dictation by carefully pronouncing all the endings.

This phenomenon is common in classrooms. It occurs every time a student can decode the teacher's expectation from the wording of the question alone, to produce a compliant answer without having to engage with its meaning. "In all didactic situations, the teacher tries to make the student know what he wants them to do, but he cannot express it in such a way that the student simply executes a series of orders. The contract functions as a system of reciprocal obligations that determines what each partner, the teacher and the student, is responsible for managing, and for which they will be accountable to the other" (Brousseau, 1986). The contract thus defines the "role of the student," as well as that of the teacher, neither of whom can substitute for the other without damaging the learning project.

For the teacher, this presents a paradox, which is difficult but necessary to manage. "The didactic contract places the teacher before a genuine paradoxical injunction : everything they undertake to make the student produce the behaviors they expect tends to deprive the student of the necessary conditions for understanding and learning the targeted concept. If the teacher says what they want, they can no longer achieve it" (Brousseau, ibid.).

Thus, in the school situation, it is as if the partners have to respect clauses that have never been discussed, clauses that are never fully respected, and whose breaches correspond to advances in shared knowledge. Because, as Brousseau says, "knowledge and the teaching project must advance under the mask."

The Didactic Contract (Continued)

Some authors prefer to call this dimension of the contract "didactic custom" (Balacheff, 1988), referring to the sociological aspect of the contract. Indeed, each class has its own custom that corresponds to the specificities of its didactic functioning. But, at the same time, and this is the core of the concept, the didactic contract serves as an important lever for the teacher, who can introduce breaks in the contract to move the didactic situation forward, to "enlist" the students in the task at hand (see Bruner's "scaffolding" function, 1983), and to succeed in devolving the problem. The didactics of mathematics thus describes, under the name of the theory of didactic situations, a dynamic that allows for the transformation, through successive shifts and ruptures, from initial "action" situations to "formulation," then to "validation," and finally to "institutionalization." The contract is thus evolutionary during the activity, so that "the theoretical concept in didactics is not the contract (the good, the bad, the true, or the false) but the process of searching for a hypothetical contract" (Brousseau, ibid.).

It is worth noting that some authors tend to dogmatize the use of the concept, especially when they see it too mechanically as something that "weighs" on the didactic structure, something the teacher cannot escape. They often fall into the temptation of using the epistemological foundations of the didactic contract to criticize innovative pedagogical proposals that, while perhaps generous, are considered naïve and disorganized. A theoretical tool essential to understanding didactic situations then becomes the instrument of a discreet restoration of magistrality, which seems all the more legitimate as it appears as the objective result of a didactic science. This would be just a new iteration of the old (and still disappointed) desire to make pedagogy "scientific." However, it first

needs new frameworks that shed light on, without ever dictating, didactic decision-making processes.

The Didactic Contract and Traditional Teaching

Students, who have been accustomed to a classic didactic contract for years, are ready to accept formal, dogmatic teaching that is relatively devoid of meaning and quite boring, as long as the teacher guarantees that the group's average results will be grouped around a "not too low" average : this is the contract, and it is what usually happens in most schools.

The experiment conducted by the IREM of Grenoble (Institute for Research in Mathematics Education) in 1980 on the "Captain's Age" problem perfectly illustrates the nature of the didactic contract : Teachers proposed the following problem to students in grades CE1 and CE2 (ages 8-9) : "... On a boat, there are 26 sheep and 10 goats. What is the captain's age ?"

This problem, posed to 97 students from CE1 and CE2 classes in French elementary schools, generated 76 responses, all of which gave the "captain's age" using the given data.

The Didactic Contract (Continued)

The children, when questioned later, are fully aware of the inconsistency of the problem, but the classic didactic contract does not expect them to question the relevance of the problem.

> "The contract does not include in the student's task that they should control the contractual legitimacy of the contract proposed to them. A student in CE1 (age 7-8) is given the following problem : 'You have 10 red pencils in your left pocket and 10 blue pencils in your right pocket. How old are you ?' The child responds, '20 years old.' When reminded that they know they are not 20 years old, the child replies, 'Yes, but it's your fault, you didn't give me the right numbers.'" (2)
>
> Given these results, it seems necessary to replace this "bad" contract with a "good" contract.
>
> This is not so simple, says Chevallard, because the teacher's role is to guide the student from a lay culture full of meaning and concrete problems to a scientific culture where concrete problems give way to abstract, theoretical problems. These must be considered in their specific framework, and the student must learn to pose and solve them independently, without considering their apparent lack of purpose.
>
> "The neglect in the didactic contract regarding the problem of relevance is precisely the price that must be paid to pull the child away from mundane play. The didactic contract is then nothing more than this extraordinary lever that allows us to shift from one world to another, to move to islands of scientific rationality, which are part of a culture that is not naturally given to us, but which must be reconstructed with each generation." (3)

The Didactic Situation

The didactic situation consists of a model of the environment (the teaching framework), and it exists whenever a teaching situation can be characterized.

Different types of situations are distinguished :

— **Action** : Corresponds to the student's production based on an implicit model.

— **Formulation** : The implementation of the explicit model.

— **Validation** : The implementation of proof mechanisms.

— **Institutionalization** : Knowledge becomes a cultural reference.

There is also the **a-didactic situation**, where the intention to teach is hidden from the student. Teaching consists of provoking the projected learning by placing the student in appropriate situations to which they will respond "spontaneously" with adaptations.

The didactic situation exists whenever the intention to teach knowledge from a teacher to a student can be characterized, and socially defined mechanisms are instituted for this purpose. The a-didactic situation concerns the hidden intention.

Modeling the Didactic Situation

In the context of the "Theory of Situations," Brousseau models "situations" as "games" (G. Brousseau, 1987, *Études en didactique des mathématiques*, IREM, University of Bordeaux I, p. 43) : "Modeling a teaching situation consists of producing a specific game of the targeted knowledge, between the subsystems : the educational system, the student system, the environment, etc." He clarifies (Brousseau, 1990, p. 315) that : "A situation is an association of 'Game-Player, Knowledge' interaction."

Various types of "situations," such as "action" situations, "formulation," "validation," and "institutionalization," have been distinguished.

To conclude this brief presentation, we can add that, initially created to make sense of observations, "situations" have become tools used both for designing and analyzing teaching sequences within the framework of didactic engineering. Thus, didactic engineering and the theory of situations are intimately associated in a dual theoretical and practical perspective.

A systemic approach allows us to consider that didactic situations stage a student, knowledge, and an "environment." Didactic situations, characteristic of the school environment, necessarily involve an intention to subject students. This subjugation can take various forms and even be hidden. This is the case in a-didactic situations. Thus, "Teaching consists of provoking the projected learning in the student by placing them in appropriate situations to which they will respond 'spontaneously' with adaptations." (Brousseau, 1990, p. 323)

By contrast, when the subject is removed from the school world, they are confronted with an "a priori" environment that lacks any didactic intentions towards them. The situations they face can then be classified as "non-didactic." "The final objective of learning is for the student to be able to use this knowledge in situations where the teacher will have disappeared." (Brousseau, 1990, p. 322)

Problem Situation

SEQUENCE : This term has two meanings : a "macro" sense when used in the context of designing a training activity, where it is referred to as a training sequence ;

and a "micro" sense when used in the context of analyzing the teaching act, where it is referred to as a pedagogical sequence.

— **Training Sequence** : The specific objectives of a training program are defined based on educational goals, general objectives, and intermediate objectives at different levels. A training sequence corresponds to the pedagogical treatment of an intermediate objective, meaning a training duration of 30 to 90 hours depending on the type of training (see Objective).

— **Pedagogical Sequence** : In the "micro" sense, meaning within the context of analyzing the teaching act, Postic and De Ketele define the sequence as follows : "A series of pedagogical acts and exchanges between the teacher and their students aimed at achieving a given goal, which fits into an overall approach. Each sequence has its own specific unit by the goal it seeks to achieve and is a step in an overall progression towards one or more objectives of the pedagogical activity." (1)

The types and sizes of sequences vary. For a long time, a simple sequence was considered as a "question, answer, feedback" sequence. However, the sequence can be longer and thus more complex. Postic and De Ketele also cite the following sequence examples in the same work :

— Introducing the subject by creating motivation through the establishment of an emotional and cognitive climate for preparing the activity, posing the problem (defining, describing, introducing facts, rules, etc.),

— Discovering a phenomenon from an observation or an experiment,

— Exploiting the facts collected, etc. (2)

Example :

Any attempt to prepare a "lesson" is an attempt at planning pedagogical action, with the "lesson" being the basic unit common to the sequence, module, curriculum, etc. Educational technology, instructional design theorists, and pedagogical engineering, all set as a basic rule the rationalization of the teaching process (needs analysis, determination of objectives, feedback regulation, remediation) to significantly improve learning.

Thus, planning the pedagogical intervention requires the anticipation and programming of different, successive, and coherent stages in which "teaching events" alternate logically and coordinately with "learning events" (see these terms). It implies that the trainer preemptively asks a number of questions necessary to clarify their intervention. These questions are, of course, "propositional," which can inspire without rigidity.

Questions a Trainer Should Ask When Preparing a Lesson :

Note :

— If you are a "busy reader," only read the bolded titles.

— If you are looking for more detailed development, read everything !

1. IN WHAT PEDAGOGICAL PROGRESSION DOES THIS LESSON FIT ?
   — Is it a continuation, a break, or a change ?
   — Is the conceptual domain already known to the students ?
   — Is it a new subject ?
   — Has it been taught before, and how ?
   — At what level ?
   — Why are we teaching this content ?

2. WHAT "TYPE" DOES THIS LESSON CORRESPOND TO ?
   — Is it a workshop lesson, a theoretical course, an application exercise, training, a review, etc. ?
3. WHAT SHOULD MY STUDENTS BE ABLE TO DO BY THE END OF THE LESSON (OR SEQUENCE) THAT THEY COULDN'T DO AT THE BEGINNING ?
   — What specific behavior should they demonstrate ?
   — What is the underlying cognitive skill ?
   — At what level : knowledge, mastery, transfer, contextualized application, or decontextualized ?
4. IS THE LESSON'S OBJECTIVE RELEVANT ?
   — Is it reasonable to spend one to four hours of class time attempting to achieve this objective, considering the students' level, the general objectives, the total duration of the training, and the available resources ?
5. WHAT TYPE OF LEARNING DOES THIS OBJECTIVE CORRESPOND TO ?
   — Is it learning facts, information, concepts, principles, cognitive strategies, procedures ? Is it a specific skill ? What mental activities does this lesson specifically engage ?
6. WHAT LEARNING SITUATIONS WILL I CREATE TO ACHIEVE THIS OBJECTIVE ?
   — What tasks can we propose to students to reach our objective ? What setup should I design to teach this content : frontal pedagogy, small groups, or a combination of both ? Should the activity center around student interactions or individualized research ?
   — What are the students' spontaneous attitudes toward the subject ? Should we vary activities and take into account individual interests or cognitive styles to ensure the participation of everyone ?
7. HOW WILL I KNOW IF MY STUDENTS HAVE OR HAVE NOT ACHIEVED THE OBJECTIVE ?
   — What form will the evaluation take ? Should it be "integrated" and formative ? Are the evaluation criteria known to the students ? Could we not discuss them beforehand and come to an agreement before the summative evaluation ?
8. ARE THERE ANY PREREQUISITES ?
   — If so, what are they ? How will I check them ? What should I do if the level is not correct ?
9. HOW WILL I GRAB THEIR ATTENTION AT THE BEGINNING OF THE LESSON, WHAT WILL THE PROBLEM BE ?
   — What points of support can I rely on ? What social practices can I refer to in order to engage students : their experiences, interests, projects, or existing knowledge ?

Additional Questions a Trainer Should Ask When Preparing a Lesson :

1. WHAT ARE THE LOGICAL STEPS OF THE LESSON ?

— The logic of learning is different from the logic of teaching : what is the most effective coherence to imagine in this case to facilitate learning ? Could we choose a less academic approach but a more effective one ?

2. HOW WILL I KNOW THAT MY STUDENTS HAVE REACHED THE EXPECTED POINT ?
   — If the steps of the lesson are numerous and delicate, what are the sub-goals (for the teacher and for the student) to achieve ?
3. WHAT SUPPORT MATERIAL SHOULD I CHOOSE TO REMAIN RELEVANT TO THE SUBJECT AND THE TYPE OF LEARNING AIMED AT ?
   — Should I produce multiple work materials or focus on one in particular ? Experience, role-playing, computing, simulation, case study, written documents, etc.

This list is not exhaustive, of course. It suggests a "directional" approach to action by building a framework of essential questions for constructing an intervention, and serves as a guide for novice trainers.

Experienced teachers, on the other hand, have widely integrated this checklist into their know-how, and their approach to preparation is more automated. Their "expertise" removes the rigid nature of planning and favors flexibility and creativity. Armed with solid preparation (whether formal or informal), they can face new variables : class dynamics, various resistances, unexpected questions... The feedback is sometimes so unexpected that they don't hesitate to redirect the action and "change course." This autonomy and independence from the program, this control due to prior experience with similar situations, is not innate. It is learned. The first step toward this mastery is indeed the preparation of the lesson, which builds confidence and courage.

Steps for Preparing a Lesson

1. Determine the objective.
2. Situate it within the general progression.
3. Determine the content and key points (see Objective-obstacle).
4. Verify the relevance of the objective.
5. If necessary, create the corresponding evaluation tool (the device) and test it.
6. Identify prerequisites.
7. Design a motivational framework for the subject and for the entire duration of the lesson.
8. Determine the type of learning targeted (concept, principle, etc.).
9. Define the micro-objectives (logical steps).
10. Define the corresponding student outputs.
11. Design micro-strategies to achieve the micro-objectives (small group work, individualized work, problem-solving, presentation, determining the level of guidance...).
12. Prepare relevant examples, teaching aids, and experiments that match the planned task and course objective.

Once again, this list of operations is not exhaustive. This approach may seem very linear, mechanistic, and overly simplistic. However, it challenges the trainer regarding their responsibility as a "facilitator" of learning. Saturated with...

Problem Situation :

A problem situation is a pedagogical situation designed by the teacher with the goal of :

— Creating a space for reflection and analysis for the students around a problem to solve (or an obstacle to overcome, according to Martinand's terminology),

— Allowing students to conceptualize new representations on a specific topic from this problem space.

Problem Solving :

Problem-solving is considered a higher intellectual activity, regarded by many authors as the most complex of cognitive activities because it engages all the intellectual faculties of the individual.

Indeed, in problem solving, the individual uses memory, perception, reasoning, conceptualization, and language, and also engages their emotions, motivation, self-confidence, and ability to control the situation.

It is said that an individual is placed in a problem-solving situation when they are confronted with a situation they have never encountered before and are trying to master it.

> "A problem does not qualify a task but a situation. [...] A problem is the representation that a cognitive system constructs from a task without immediately having an admissible procedure to achieve the goal. The construction of the..."

Problem Situation (Continued)

The representation of the task is called understanding, and the construction of the procedure is the strategy for solving the problem. (1)

Of course, not all situations constitute a "problem" for every individual. With remarkable conciseness, Richard clarifies the essence of the issue :

> "It is important to carefully distinguish between execution activities and problem-solving activities... This distinction cannot be made by simply considering the tasks : some tasks are problems for certain subjects and execution situations for others. [...] What defines a problem or an execution situation is not only the situation itself, but [...] the relationship between the task and the competencies of the individual. Therefore, it is important to clarify how a situation becomes a problem for the individual and how a problem situation becomes an execution situation." (2)

EXAMPLES OF PROBLEM SITUATIONS (6) :

— **Example 1** : Repairing a broken device when you don't have the assembly diagram and only have a vague idea of how the device works.

— **Example 2** : Maier's Horse. A farmer goes to the market. He buys a horse for 6,000 francs. An hour later, he resells it for 6,000 DA. A few minutes later, he buys it back for 8,000 DA and resells it for 9,000 DA half an hour later. Has he earned 1,000 DA ? 3,000 DA ? 4,000 DA ? Nothing at all ? Has he lost money ?

— **Example 3** : Making seven equal triangles with nine matchsticks.

When faced with a problem, the individual first constructs a representation of the problem (understanding), and from this representation, the initiation of problem-solving procedures depends.

> "A situation that is a problem becomes an execution situation when, for this specific context, we have constructed the prerequisites of the actions that allow us to achieve the sub-goals, and we also have the heuristics and the necessary knowledge to define the sub-goals." (3)

The construction of a representation depends on several factors : first, perceptual (recognition of a category of problems), then motivational (previous successful experiences in this category of problems).

— The factors related to perception can be decisive : the indices or stimuli may not be sufficient to categorize the problem and call on a known procedure. The influence of the context is predominant here, and psychologists of form refer to "fixation" when an individual cannot change their hypothesis to reason in a different framework (such as the problem of Maier's 9 points), which affects both the cognitive and emotional aspects. (4) When we are able to create a problem, we are generally able to solve it.

Rimoldi's Experiment :

Rimoldi argues that the main difficulty in solving a problem lies in choosing the right information, that is, the information that is relevant for solving the given problem. He therefore creates situations designed to improve an individual's ability to choose the relevant information. The experimenter begins by proposing a problem whose resolution requires the subject to actively seek information. This information will be provided by the experimenter in response to the questions the subject chooses to ask. All the questions are written on cards, with the corresponding information on the back. The subject chooses the question they wish to ask and flips the card to obtain the answer. The information provided can take various forms depending on the nature of the problem being addressed : a short text, a drawing, laboratory analysis results, an X-ray (for the study of medical diagnostics).

Here is an illustration of the type of situation proposed by Rimoldi :

Problem : Bill is a student working during his vacation. The company that employs him pays some of its workers 45 dollars per week, others 65 dollars per week, and others 90 dollars per week. You need to find out how much Bill earns per week.

Instructions : You can find the solution to this problem by selecting questions from the list provided to you. First, read all the questions, then decide which one you want answered first. Take the card with that question on it, flip it over, and you will find the answer. Then, decide on the second question you want to ask. Repeat this process until you have discovered the solution to the problem. Select as many questions as you deem necessary to solve the problem. Don't hesitate to ask a question you think is necessary, but don't ask more questions than you need. Remember, you need to find out if Bill earns 45 dollars, 65 dollars, or 90 dollars per week.

List of Questions :

— Who is Bill ?
— Does Bill do manual work ?
— How old is Bill ?
— Does his work require a lot of experience ?
— Does Bill work outdoors ?
— Does Bill work on a truck ?
— Do manual workers earn 45 dollars per week ?
— Do young people aged 18 usually earn 45 dollars per week ?
— Do experienced workers earn 45 dollars per week ?
— Do manual workers earn 65 dollars per week ?
— Do young people aged 18 earn 65 dollars per week ?
— Do manual workers earn 90 dollars per week ?
— Do young people aged 18 earn 90 dollars per week ?
— Do experienced workers earn 90 dollars per week ?
— Do workers earning 45 dollars per week do overtime ?
— Do workers earning 65 dollars per week do overtime ?
— Do workers earning 90 dollars per week do overtime ?

List of Information Provided by the Questions :

— Bill is a student who works during his vacation. He earns either 45 dollars, 65 dollars, or 90 dollars per week.
— Yes, Bill does manual work.
— Bill is 18 years old.

Answers to the Questions in Rimoldi's Experiment :

1. No, Bill's work does not require much experience.
2. Yes, Bill works outdoors.
3. No, Bill does not work on a truck.
4. Manual workers who work on trucks earn 45 dollars per week.
5. No, young people aged 18 do not earn 45 dollars per week.
6. No, experienced workers do not earn 45 dollars per week.
7. Yes, manual workers earn 65 dollars per week.
8. Yes, young people aged 18 earn 65 dollars per week, provided they work outdoors.
9. No, manual workers do not earn 90 dollars per week.
10. No, young people aged 18 do not earn 90 dollars per week.
11. Yes, experienced workers earn 90 dollars per week.
12. Yes, workers who earn 45 dollars per week sometimes work overtime.
13. No, workers who earn 65 dollars per week do not work overtime.
14. Yes, workers who earn 90 dollars per week work overtime if they are 18 years old.

Teaching Problem-Solving Skills

Teaching the ability to solve problems is a difficult and complex subject that researchers have been working on for a long time, without being able to significantly determine whether certain strategies are more effective than others.

III. Research Methodology in Didactics

Like many research disciplines, didactics has two dimensions :

1. **Research for knowledge**
2. **Research for decision-making**

In the first case, it is about studying processes, particularly student and teacher behavior (attitudes, conceptions, strategies, and the assessment of knowledge and skills).

In the second case, the goal is to better understand the conditions, constraints, and consequences of decisions related to the curriculum (programs, types of activities, modes of evaluation, etc.).

Knowledge Acquisition and Curriculum Decisions

The study of learning behaviors has demonstrated that knowledge is not simply transmitted but must be reconstructed and appropriated by the learner. The metaphor of "transmission" loses its value as soon as we focus on learning mechanisms. This realization has led to increasing interest in research that explores how learners acquire knowledge, particularly by examining aspects related to psychological development and the pedagogical mediations employed.

The use of computer simulations is expected to open new possibilities for testing and refining learning models. Studies on curriculum decisions—both from an external perspective (history and sociology) and an internal perspective (research-intervention)—have highlighted the conditions and unique aspects of what can truly be called educational creativity in terms of content development. Reference knowledge and practices, compromises between educational goals, and didactic transposition are now more clearly understood.

Didactic research, whether "scientific" or "technological," has led to the construction of key concepts. Some of these concepts have transcended their original didactic fields, including :

— **Didactic transposition**
— **Reference practices**
— **Didactic contract**
— **Conceptual field**
— **Learner conceptions**
— **Objective-obstacle**

These concepts serve as foundational elements for theoretical frameworks.

Didactic Research Methods

From a methodological standpoint, didactic research does not have a unique specificity. Its techniques and instruments belong to the broader fields of human and social sciences and cultural technology. Fieldwork plays a crucial role, allowing researchers to differentiate between the perspectives of university discipline specialists and the conclusions drawn by psychologists, sociologists, historians, and philosophers regarding teaching, learning, and curriculum reforms.

Among the disciplines that offer particularly valuable interactions for didactic research, the following stand out :

— **Cognitive sciences** (exploring the origins of learners' conceptions, problem-solving processes, and learning environments)

— **History of education** (especially the history of academic disciplines)
— **Educational technology**

Didactics of Trainer Training

Note :

It is important to remember that the didactics of trainer education is distinct from the didactics of student education and remains an area still in development.

When considering trainer education and the **didactic triangle**, the focus should not be on the vertices of the triangle but rather on its sides.

One could imagine a second triangle, superimposed on the first, defining the key focal points of trainer education :

— Understanding the challenges associated with teaching a subject
— Evaluating teaching practices
— Addressing fundamental questions, such as :
  — How can we train trainers to recognize and understand learners' difficulties ?
  — How can trainers take into account the specific challenges faced by their learners ?
  — How can they assess the effectiveness of their didactic choices ?

It is recommended that trainer education programs be structured around the following key elements :

— **a)** Identifying learning obstacles
— **b)** Observing didactic situations

c) Didactic Engineering

Didactic engineering refers to the creation of a **teaching sequence** where instruction is planned in meticulous detail.

d) Evaluation

Exercise : Decimal Numeration in Cycle 2 – Stamp Problem

A teacher gives the following evaluation exercise at the end of the first trimester to a **CE2** class :

> Stamps are sold in booklets of 10. How many booklets must be purchased to stamp 254 envelopes ?

1. What is the underlying mathematical concept ?

Even though this problem involves **division** (quotient by excess), dividing **254 by 10** is equivalent to finding the number of tens in **254**, making this exercise primarily an **exercise in numeration**.

2. Analysis of Five Student Responses

Student	Calculation and Strategy	Response Expression	Knowledge, Errors, and Gaps
Sonia	Decomposes 200 as **20 × 10**, then decomposes **54** additively into tens and units. Counts the tens based on these two decompositions.	Gives an answer related to the question : the number **74** represents the booklets needed, even though her counting is incorrect.	The equation **200 = 20 × 10** (with 20 written below) shows that S. correctly counted **20 packs of ten**. However, when counting the tens in **54**, she mistakenly counts units instead of tens. This error is common in Cycle 2 when there is a unit change—here, the unit is no longer an envelope but a **pack of 10 envelopes**.
Aurélie	Decomposes 100 into a sum of 10s, multiplies by 2, then breaks **54** into **5 booklets of 10** and **4 leftover stamps**.	Does not answer the question directly ; expresses the number of stamps in booklets and individual stamps (**25 booklets plus 4 stamps**).	A. does not fully grasp numeration beyond **100**—she cannot directly determine the number of **tens** in **200**. There is also a counting error in steps of 10 (**she wrote 70, which is a partial total she calculated**).
Hélène	Decomposes **254** into **200 + 54** and directly assigns the correct number of booklets to each part.	Gives the correct answer and highlights it by circling it.	H. relies on numeration (**200 = 2 × 10 packs of 10 = 20 packs of 10**). She understands that **6 booklets** are needed for the **54 stamps** and even writes the remaining stamps (**"6t" meaning 6 stamps**).
Laurent	Uses repeated addition of booklets until reaching **254 stamps**.	Does not write the final answer (though his method would allow him to determine it).	L. approaches the problem in a **practical way**, like a shopkeeper. He explains **how to obtain 254 stamps using booklets of 10**.
Jean	Decomposes **254** into **200 + 50 + 4** and assigns the correct number of booklets to each part.	Does not explicitly answer the question, but understands that **26 booklets** are needed since he states that for the **last 4 stamps**, **1 booklet is required**.	J. does not use additive or multiplicative notation but demonstrates **good numeration skills** by finding the number of **tens** in **200** and **50**. Like Hélène, he also indicates the **unused stamps (6)**.

3. Do these students have the expected mastery of decimal numeration for their grade level ?

None of the students fully **master** our positional number system well enough to **directly determine** the number of tens in **254** from its numeric representation.

30 An Example of Mathematical Production

Geometric Proofs of Pythagoras

Pythagoras' Theorem

In any right-angled triangle, the sum of the squares of the lengths of the sides forming the right angle is equal to the square of the length of the hypotenuse, and vice versa.

Pythagoras' theorem is very popular, and everyone remembers $a^2 + b^2 = c^2$, with Euclid's formulation :

> "the square of the hypotenuse is equal to the sum of the squares of the other two sides,"

which I prefer over the encyclopedic formulation :

> "the square of the length of the hypotenuse, which is the side opposite the right angle, is equal to the sum of the squares of the lengths of the legs."

It can be found on a cuneiform tablet from Babylon, around 2000 BC.

Pythagoras of Samos (569–494 BC), a mathematician of Ancient Greece, did not discover this theorem but presented it graphically and does not seem to have proven it.

It was not until Euclid, around the 3rd century BC, that the first known proof appeared.

The American mathematician Elisha Scott Loomis cataloged more than 300 proofs ; about ten of them are presented here.

Four Types of Pythagorean Proofs

— Proof from Euclid's *Elements*, quite complex, and no longer taught today.

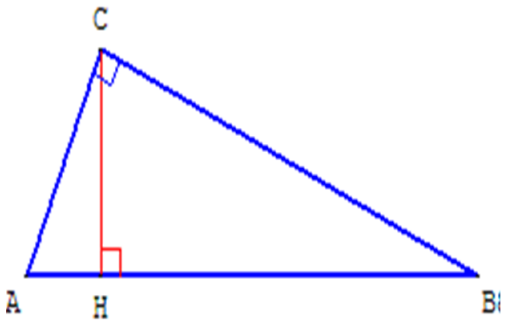

FIGURE 2 – Description de l'image

II. Proof using the Area Method

Based on the similarity of the large right-angled triangle ABC with the right-angled triangles ACH and BCH formed by the legs and the height CH dropped onto the hypotenuse :

The area of the large triangle is the sum of the areas of the two smaller ones.

For similar right-angled triangles, their areas are proportional to the squares of their hypotenuses ; thus, the square of the hypotenuse of the large triangle is equal to the sum of the squares of the hypotenuses of the two smaller triangles.

III. Proof by Complementarity

Based on area equalities using puzzle-like manipulations, accessible starting from the end of primary school (Cycle III).

IV. Arithmetic Proof

Calculating the areas of different squares (Bhaskara).

Converse of Pythagoras' Theorem

Euclid's Elements, Book I, Proposition 48 :

If, in a triangle, the square of the longest side is equal to the sum of the squares of the two other sides, then the triangle is right-angled.

The longest side is the hypotenuse, and the right angle is the angle opposite to the hypotenuse.

Note : As in Euclid's presentation and to simplify memorization, the term "length" is often omitted on this page ; each side is then identified with its length.

1. Pythagoras' Proof by Euclid
— 1.a. Euclid's Elements, Book I, Proposition 47
— 1.b. The so-called "windmill figure"
— 1.c. First theorem of Euclid
— 1.d. Splitting a hexagon into two
2. Proof of Pythagoras' Theorem via Pappus
— 2.a. Clairaut via Pappus
— 2.b. Clairaut's theorem
— 2.c. Right-angled triangle BOA
— 2.d. Chinese puzzle
3. Proof of Pythagoras by Leonardo da Vinci (1452–1519)
4. Pythagoras by Renan 5. Bhaskara's Construction
6.a. The Problem of President James Abraham Garfield
6.b. The Four Triangles – Chinese Puzzle
6.c. Application : Three Contiguous Squares
7. Thabit Ibn Qurra
7.a. Clairaut
7.b. Pythagoras' Square Puzzle
7.c. Square Puzzle (Liu Hui, China, 3rd century)
8. Périgal's Puzzle
8.a. Cutting Squares
8.b. Pythagoras' Puzzle in Five Pieces
9. Similar Triangles in a Circle
10. Similar Triangles in a Rectangle 11. Lunules 11.a. Lunules of Hippocrates of Chios
11.b. Generalized Pythagorean Theorem

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