BLIND ESTIMATION OF LONG IMPULSE RESPONSE AND NON-MINIMUM PHASE WAVELETS APPLICATION TO SEISMIC DATA

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ABSTRACT

In seismic deconvolution, blind approches must be considered in situations where the reflectivity sequence, the source wavelet signal and the noise power level are unknown. In the presence of long, non minimum-phase, source wavelets, strong interference of the reflectors contributions make the wavelet estimation and deconvolution procedure from recorded data complicated. In this paper, we address this problem in a two steps approach. First, a robust but truncated estimate of the wavelet is performed using a standard maximum likelihood approach. Then improved wavelet estimation is achieved by fitting an ARMA model to the initial MA wavelet by using the Prony algorithm. The algorithmic problem of wavelet initialization is also addressed. Simulation results and real data experiments show the significant improvement brought by this approach.

1. INTRODUCTION

This paper deals with seismic deconvolution, which aims at recovering the geological structure of the underground sedimentary layers from seismic data records [13]. As usualy in this kind of situation, the seismic traces are modelled as the output of a filter that represents the transmitted wavelet, with an input consisting in a two components Gaussian mixture: the Gaussian component with high variance models the strong reflectivity at layers interfaces [1]. Recovering this sequence from the data enables detecting the layers position in the subsurface.

In some experiments of practical interest the wavelet is quite long [13]. In such situations, estimating the model parameters generally yields a high variance of the wavelet estimator. In this paper, we propose a new method that permits to overcome this problem within the framework of classical blind seismic deconvolution techniques. More precisely, a two steps approach is proposed: the first step yields a robust Maximum Likelihood (ML) estimate of a truncated version of the wavelet, via a Stochastic Expectation Maximization (SEM) approach. Then, an improved wavelet estimation is achieved by fitting an ARMA model to the initial MA wavelet by means of a Prony algorithm. Furthermore, a new criterion is also proposed for accurate estimation of the wavelet impulse response maximum position, which is an important algorithmic issue for accurate wavelet estimation. The paper is organized as follows: Section 2 describes the data model, while section 3 is devoted to initial estimation of the parameters. In section 4, improved wavelet estimation is considered. Finally, in section 5 we check on simulation and real data experiments the significant improvement brought by this approach. A conclusion is presented in section 6.

This work was supported by IFREMER

2. DATA MODEL

The observed signal $\mathbf{y} = (y_k)_{k=1,N}$ is of the form

$$y_k = \sum_{i=0}^{L} h_i r_{k-i} + w_k, \tag{1}$$

where $\mathbf{h} = (h_k)_{k=0,L}$ is the wavelet finite impulse response column vector of length L, $\mathbf{r} = (r_k)_{k=1,N}$ is the reflectivity sequence, and $\mathbf{w} = (w_k)_{k=1,N}$ is the observation noise sequence, with variance σ_w^2. The reflectivity process $\mathbf{r}$ is described by a generalized Bernoulli-Gaussian process [1], characterized by an underlying state model $\mathbf{q} = (q_k)_{k=0,N}$, with $q_k = 1$ at high reflectivity points and $q_k = 0$ at low reflectivity points. The corresponding reflectivity r_k is distributed according to a zero mean Gaussian distribution with variance σ_1^2 if $q_k = 1$ or σ_0^2 if $q_k = 0$,

$$\begin{array}{c} p(q_k = 1) = 1 - p(q_k = 0) = \lambda \\ r_k \sim \lambda\mathcal{N}(0, \sigma_1^2) + (1-\lambda)\mathcal{N}(0, \sigma_0^2), \end{array} \tag{2}$$

where λ is the probability of having a reflector at a given position and $\sigma_1^2 \gg \sigma_0^2$.

3. INITIAL PARAMETER ESTIMATION

We address the blind deconvolution problem through the classical Maximum Likelihood criterion [3] which leads to calculate

$$\hat{\theta}_{\mathbf{MV}} = \arg\max_{\theta} ln(p(\mathbf{y}|\theta)), \quad (3)$$

where θ is the parameter vector of interest. Here, $\theta = (\mathbf{h}, \lambda, \sigma_0^2, \sigma_1^2, \sigma_w^2)$.

In fact, we are faced to an incomplete data problem [3, 6], where the incomplete data are given by $\mathbf{z} = (z_k)_{k=0,N}$ and $z_k = (q_k, r_k)$. The joint probability density is expressed by

$$p(\mathbf{y}, \mathbf{z}|\theta) = p(\mathbf{y}|\mathbf{z}, \theta)p(\mathbf{z}|\theta). \quad (4)$$

As $\mathbf{z} = (\mathbf{q}, \mathbf{r})$, it can also be written

$$p(\mathbf{y}, \mathbf{z}|\theta) = p(\mathbf{y}|\mathbf{z}, \theta)p(\mathbf{r}|\mathbf{q}, \theta)p(\mathbf{q}|\theta). \quad (5)$$

Each component of this equations can be easily expressed. As $\mathbf{q}$ is a vector of independent Bernoulli variables,

$$p(\mathbf{q}) = \prod_{k=1}^{N} p(q_k), \quad (6)$$

with $p(q_k|\theta) = \lambda^{q_k}(1-\lambda)^{1-q_k}$. Furthermore, since the r_k are independent, conditional to the variables q_k

$$\begin{array}{ll} p(\mathbf{r}|\mathbf{q}, \theta) & = \prod_{k=1}^{N} p(r_k|q_k, \theta) \\ p(\mathbf{y}|\mathbf{z}, \theta) & = \frac{1}{(2\pi\sigma_w^2)^{N/2}} \exp -[\frac{\sum_{k=1}^{N}(y_k - h * r_k)^2)}{2\sigma_w^2}]. \end{array} \quad (7)$$

Then, up to a constant, the complete log-likelihood is expressed by:

$$\begin{array}{l} L(\mathbf{y}, \mathbf{z}|\theta) \propto -\sigma_w^{-2}(\mathbf{y} - \mathbf{Hr})^T(\mathbf{y} - \mathbf{Hr}) - N.ln(\sigma_w^2) \\ \quad -\sigma_1^{-2}\mathbf{r}^{\mathbf{T}}\mathbf{Q}\mathbf{r}(\mathbf{z}) - \sigma_0^{-2}\mathbf{r}^{\mathbf{T}}(I - \mathbf{Q})\mathbf{r}(\mathbf{z}) \\ \quad +2\mathbf{q}^{\mathbf{T}}\mathbf{q} ln(\sigma_1^{-1}\lambda_1) + 2(N - \mathbf{q}^{\mathbf{T}}\mathbf{q}) ln(\sigma_0^{-1}(1-\lambda)), \end{array} \quad (8)$$

where $\mathbf{Q} = \text{diag}(\mathbf{q})$ and $\mathbf{H}$ is the convolution matrix associated with $\mathbf{h}$. Note that $\mathbf{Hr} = \mathbf{Rh}$, where $\mathbf{R}$ is the convolution matrix associated with $\mathbf{r}$. Then, when the complete data vector $\mathbf{z}$ is known, the derivation of $\hat{\theta}_{\mathbf{MV}}$ is straightforward; for fixed $(\mathbf{y}, \mathbf{z})$, the ML estimator of θ is obtained from (8):

$$\begin{array}{ll} \hat{\mathbf{h}} & = (\mathbf{R^T R})^{-1}\mathbf{R^T y}, \ \hat{\lambda} = N^{-1}\mathbf{q^T q} \\ \hat{\sigma}_w^2 & = N^{-1} \| \mathbf{y} - \mathbf{R}\hat{\mathbf{h}} \|_2^2 \\ \hat{\sigma}_0^2 & = \frac{\mathbf{r^T}(I-\mathbf{Q})\mathbf{r}}{N - \mathbf{q^T q}}, \ \hat{\sigma}_1^2 = \frac{\mathbf{r^T Q r}}{\mathbf{q^T q}}. \end{array} \quad (9)$$

where $\| \cdot \|_2$ represents the quadratic norm.

In practice, $\mathbf{z}$ is not known. In such a situation, a SEM (Stochastic Expectation Maximization) algorithm, involving simulation of $\mathbf{z}$, can be used to solve the estimation problem [9]: starting from initial values $\theta^{(0)}$ and $\mathbf{z}^{(0)}$ for θ and $\mathbf{z}$ respectively, the SEM i^{th} iteration is of the form

- SE step: simulate $\mathbf{z}^{(i)} \sim p(\mathbf{z}|\theta^{(i-1)})$
- M step: estimate $\theta^{(i)}$ according to eq. (9).

The expression of $p(\mathbf{z}|\theta^{(i-1)})$ can be found in [6]. In particular, $p(\mathbf{r}|\mathbf{q}, \theta^{(i-1)})$ is of the form $(1-q_k)\mathcal{N}(m_0, \sigma_0^2) + q_k\mathcal{N}(m_1, \sigma_1^2)$. In practice, $\mathbf{z}$ can be simulated using a Gibbs sampler [8]. The Gibbs sampler iteration is implemented as follows [6]:
For $k = 1, \ldots, N$,

- Compute $p(q_k = 1|\mathbf{y}; \mathbf{z}_{-k})$, where $\mathbf{z}_{-k}$ is $\mathbf{z}$ with removed k^{th} entry z_k.
- Simulate $u \sim \mathcal{U}_{[0,1]}$ ($\mathcal{U}_E$ is the uniform distribution on E) and take $q_k = \mathbb{I}_{[0, p(q_k=1|\theta, \mathbf{z}_{-k})]}(u)$ ($\mathbb{I}_A$ is the index function of A).
- simulate $r_k \sim (1-q_k)\mathcal{N}(m_0, \sigma_0^2) + q_k\mathcal{N}(m_1, \sigma_1^2)$
- update z_k: $z_k = (q_k, r_k)$.

let us remark that similar performance results could obtained if the above SEM estimation procedure is replaced by a Markov Chain Monte Carlo (MCMC) approach [14].

4. IMPROVED WAVELET ESTIMATION

In some seismic experiments the wavelet impulse response $\mathbf{h}$ is quite long. In such cases, the mean square error of the estimator is quite large. In particular, the last coefficients of $\mathbf{h}$, which have small values, are poorly estimated. For this reason, searching for a vector $\mathbf{h}$ with reduced length generally enables a good compromise between bias and variance properties of the estimator.

However, performing the deconvolution with a truncated wavelet will generate degradated performance for the reflectivity sequence.

Improved ARMA(p, q) wavelet estimation In order to improve the deconvolution performance, we assume that the MA(L) wavelet model that has been estimated by means of the SEM procedure described in the previous section is in fact a truncated version of the true wavelet, of length $L' > L$. The value of L is not much critical. Simply, the envelope of the MA(L) impulse response should not decay too much. An efficient approach for choosing L is described further in the text. Since L' can be quite large in practice and, often, the wavelet has an oscilatory shape, it can be modeled efficiently as an ARMA(p, q) impulse response. In order to estimate it from the initial MA(L) wavelet, we propose to use the Prony method [2]. More precisely, letting $h(z)$ represents the transfer function of the ARMA(p, q) model to be estimated, we have

$$h(z) = (\sum_{l=0,q} b_l z^{-l})(1 + \sum_{k=0,p} a_k z^{-k})^{-1}. \quad (10)$$

Then, the coefficients $\mathbf{a} = (a_k)_{k=1,p}$ and $\mathbf{b} = (b_l)_{l=0,q}$ are obtained by minimizing the following quadratic criterion:

$$J(\mathbf{a}, \mathbf{b}) = \sum_{l=0}^{L'} |\sum_{k=0}^{p} a_k h_{l-k} - b_l|^2, \qquad (11)$$

where we note $b_l = 0$ for $l > q$. Straightforward calculations show that $(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = \arg\min_{\mathbf{a},\mathbf{b}} J(\mathbf{a}, \mathbf{b})$ is given by

$$\hat{\mathbf{a}} = -(\mathbf{H}_1^H \mathbf{H}_1)^{-1} \mathbf{H}_1^H \mathbf{V}, \hat{\mathbf{b}} = \mathbf{H}_0 [1 \quad \hat{\mathbf{a}}^T]^T \qquad (12)$$

where

$$\begin{aligned} \mathbf{H}_0 &= \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ h_1 & h_0 & \cdots & 0 \\ \vdots & \ddots & . & . \\ h_d & h_{q-1} & \cdots & h_{q-p} \end{bmatrix} \\ \mathbf{H}_1 &= \begin{bmatrix} h_q & h_{q-1} & \cdots & h_{q-p+1} \\ h_{q-1} & h_{q-2} & . & . \\ \vdots & \cdots & \ddots & . \\ h_d & h_{q-1} & \cdots & h_{N-q} \end{bmatrix} \\ \mathbf{V} &= [h_{q+1}, \ldots, h_N] \quad , N - q > p. \end{aligned} \qquad (13)$$

In order to estimate the respective orders p and q of the AR and of the MA parts, we use a Kurtosis maximization criterion [10].

Initial MA model order selection Now, let us explain how the length L of the initial $MA(L)$ wavelet can be chosen. Denoting $\hat{\mathbf{h}}_i$ ($i = l_1, ..., l_2$) the estimated wavelet of length i and $\hat{\mathbf{h}}_{\mathbf{m}} = (l_2 - l_1 - 1)^{-1} \sum_{i=l_1}^{l_2} \hat{\mathbf{h}}_{\mathbf{i}}$, we note $\widehat{MSE}_i = \| \hat{\mathbf{h}}_i - \hat{\mathbf{h}}_m \|_2$. Comparing $\widehat{\mathbf{MSE}} = (\widehat{MSE}_i)_{i=l_1,l_2}$ and $\mathbf{MSE} = (MSE_i)_{i=l_1,l_2}$, where $MSE_i = \|\hat{\mathbf{h}}_i - \mathbf{h}\|_2$ (see Figure4), shows that a good choice for L is obtained by considering the minimum of $\widehat{\mathbf{MSE}}$.

Initialization It is well known that the non-minimum phase structure of the wavelet $\mathbf{h}$ makes its estimation complicated. In particular the wavelet estimation is not robust to initialization. A simulated annealing version of the SAEM algorithm could be used to overcome this problem [11]. Here, we propose an alternative deterministic procedure for initializing the MA(L) wavelet estimate. As in [11], we initialize $\mathbf{h}$ with the vector $\mathbf{h}^{(i)}$ ($i = 1, \ldots, k_1$) that has 0 entries, except the i^{th} one, which is equal to 1, and we perform the deconvolution for each i. Now, let $C_i = 1$ if $\hat{\mathbf{h}}_i$ is increasing at the origin and $C_i = -1$ if it decreases at the origin. It can be checked that $\mathbf{C} = (C_i)_{i=k_1,k_2}$ changes of sign at values of i corresponding to the true wavelet local optima (Figure 3). The retained solution is the one that maximizes the Kurtosis of estimated reflectivity $\hat{\mathbf{r}}$ at such points.

Deconvolution When $\mathbf{h}$ has been estimated, the last step consists in a deconvolution via an MPM approach that yields the final estimate of the reflectivity sequence [14].

5. RESULTS

In this section, we present an example for simulated and real data. Figure 1 and 2 represent the simulated reflectivity and observation ($\sigma_w^2 = 1.0^{-4}$, $\lambda = 0.1$, $\sigma_0^2 = 10^{-4}$, $\sigma_1^2 = 0.1$). The true wavelet and the function $\mathbf{C}$ introduced in the previous section are given in Figure 3, while $\mathbf{MSE}$ and $\widehat{\mathbf{MSE}}$ are given in Figure 4. Table 1 and Figure 3 show that the maximum position is correctly recovered with the proposed procedure. Figure 5 shows the improvement brought by the MA truncated wavelet + ARMA extension modelization compared to a direct estimation of the full length wavelet. Figures 6 to 9 show the results obtained for real data (Zaiango campaign, Ifremer [13]). We get respectively $\| \mathbf{y} - \hat{\mathbf{h}} * \hat{\mathbf{r}} \|_2 = 2.3$ and 0.93 for the MA(L') and MA(L)+ARMA(p, q) wavelets estimates, which shows the higher deconvolution capability of the proposed method.

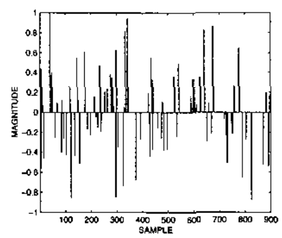

Figure 1: simulated reflectivity sequence.

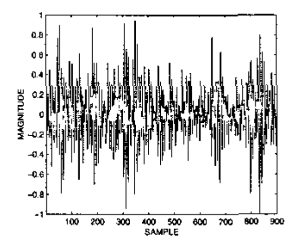

Figure 2: simulated noisy seismic data. SNR=10dB.

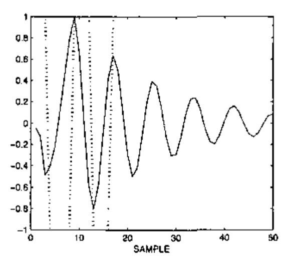

Figure 3: '- -':true wavelet '...': $\mathbf{C}$.

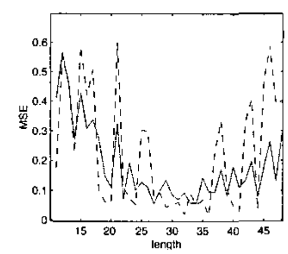

Figure 4: '- -': $\mathbf{MSE}$, '...':$\widehat{\mathbf{MSE}}$.

maximum position candidates(Fig. 3)	4	9	12	16
kurtosis	0.0079	0.0351	0.0217	0.0070

Table 1: estimated kurtosis at changes of $\mathbf{C}$.

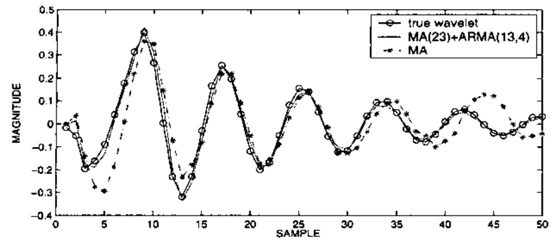

Figure 5: estimated wavelet for SNR=10dB.

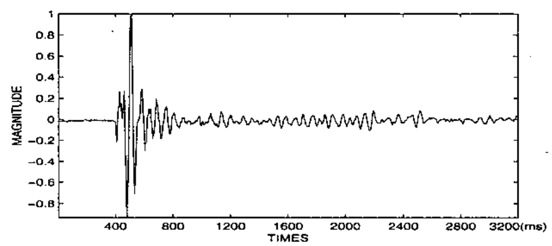

Figure 6: registered seismic trace n^o602.

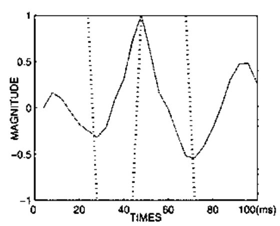

Figure 7: '- -': initial estimate of $\mathbf{h}$, '...': $\mathbf{C}$

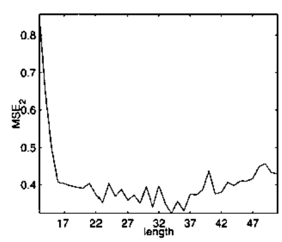

Figure 8: $\widehat{\mathbf{MSE}}$.

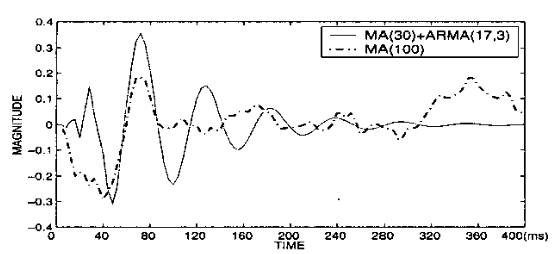

Figure 9: estimated wavelet.

6. CONCLUSION

In this paper, we have proposed a new approach for blind estimation of long impulse response and non-minimum phase wavelets.

7. REFERENCES

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