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Study and Design of Robust Observers for Nonlinear Systems: Application on an Artificial Pancreas

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ملخص

تتناول هذه الأطروحة التحدي الجوهري لتقدير الحالة للأنظمة الديناميكية غير الخطية غير المؤكدة، مع تركيز خاص على تطبيقها في تنظيم مستوى السكر في الدم لمرضى السكري من النوع الأول. من خلال صياغة فئة محددة من الأنظمة غير الخطية المثلثية، تم وضع إطار نظري صارم ينتقل من هياكل مراقبات لوينبرجر القياسية إلى مقدرات قوية قادرة على تعويض اضطرابات الوجبات غير المعروفة، وذلك عبر تطبيق نظرية استقرار ليابونوف ومتباينات المصفوفات الخطية.

تم تطوير ومقارنة ثلاث منهجيات للمراقب بشكل تدريجي: مراقب أساسي بدون تعويض للمدخلات غير المعروفة، ومراقب مع تقدير صريح للاضطرابات، ومراقب متطور يستخدم قيوداً تعتمد على الحالة. أظهر التحقق العملي لهذه المنهجيات، الذي تم إجراؤه باستخدام نموذج بيرجمان الأدنى ومحللات متباينات المصفوفات الخطية في برنامج ماتلاب، أن المراقب المتطور يتفوق باستمرار على النسخ السابقة؛ حيث حقق مسارات تقدير أكثر سلاسة لمستوى الجلوكوز، وتقارباً أسرع خلال التغيرات العابرة، وانخفاضاً في التجاوز أثناء فترات تناول الوجبات، بالإضافة إلى دقة أعلى في إعادة بناء تركيز الأنسولين في البلازما.

تم تطوير نموذج أولي وظيفي لنظام البنكرياس الاصطناعي ذي الحلقة المغلقة باستخدام متحكم دقيق إي إس بي ٢٣، ومنطق تحكم يعتمد على العتبات، وتشغيل دقيق لمحرك خطوي، مما أثبت بنجاح المبادئ الأساسية لتوصيل الأنسولين المؤتمت على منصة معيارية وفعالة من حيث التكلفة. يجسر هذا البحث الفجوة بين التحليل النظري للاستقرار القائم على المتباينات الخطية والمتطلبات العملية للاستخدام اليومي، مما يمهد الطريق لأنظمة توصيل أنسولين أكثر موثوقية ودقة وتخصيصاً.

الكلمات المفتاحية: تقدير الحالة، الأنظمة غير الخطية، السكري من النوع الأول، تنظيم السكر في الدم، مراقبات لوينبرجر، استقرار ليابونوف، متباينات المصفوفات الخطية (إل إم آي)، نموذج بيرجمان الأدنى، البنكرياس الاصطناعي، الحلقة المغلقة، متحكم إي إس بي ٢٣، محرك خطوي، نموذج أولي وظيفي.

Abstract

This thesis addresses the critical challenge of state estimation for uncertain nonlinear dynamic systems, with a particular focus on its application to glycemic regulation in Type 1 Diabetes Mellitus. By formalizing a specific class of triangular nonlinear systems, a rigorous theoretical framework is established that progresses from standard Luenberger observer structures to robust estimators capable of compensating for unknown meal-induced disturbances through the application of Lyapunov stability theory and Linear Matrix Inequalities.

Three observer methodologies are progressively developed and compared: a basic observer without unknown input compensation, an observer with explicit disturbance estimation, and an enhanced observer using state-dependent constraints. The practical validation of these methodologies, conducted using the Bergman Minimal Model and MATLAB LMI solvers, demonstrates that the enhanced observer consistently outperforms previous iterations, achieving smoother glucose estimation trajectories, faster convergence during transients, reduced overshoot during meal impulses, and higher fidelity in reconstructing plasmatic insulin peaks.

A functional prototype of a closed-loop artificial pancreas system is developed using an ESP32 microcontroller, threshold-based control logic, and precise stepper motor actuation, successfully demonstrating the fundamental principles of automated insulin delivery on a cost-effective, modular platform. This research effectively bridges the gap between theoretical LMI-based stability analysis and the practical exigencies of ambulatory use, paving the way for more reliable, precise, and personalized insulin delivery systems.

Keywords: Artificial pancreas, closed-loop system, ESP32 microcontroller, threshold-based control, automated insulin delivery, LMI-based analysis

Résumé

Cette thèse aborde le défi critique de l'estimation d'état pour les systèmes dynamiques non linéaires incertains, avec une attention particulière portée à son application à la régulation glycémique dans le diabète de type 1. En formalisant une classe spécifique de systèmes non linéaires triangulaires, un cadre théorique rigoureux est établi, progressant des structures classiques d'observateurs de Luenberger vers des estimateurs robustes capables de compenser les perturbations inconnues induites par les repas, grâce à l'application de la théorie de la stabilité de Lyapunov et des inégalités matricielles linéaires (LMI).

Trois méthodologies d'observateurs sont progressivement développées et comparées : un observateur de base sans compensation d'entrée inconnue, un observateur avec estimation explicite des perturbations, et un observateur amélioré utilisant des contraintes dépendantes de l'état. La validation pratique de ces méthodologies, réalisée à l'aide du modèle minimal de Bergman et des solveurs LMI de MATLAB, démontre que l'observateur amélioré surpasse systématiquement les itérations précédentes, atteignant des trajectoires d'estimation de la glycémie plus fluides, une convergence plus rapide lors des transitoires, une réduction du dépassement lors des impulsions de repas, et une plus grande fidélité dans la reconstruction des pics d'insuline plasmatique.

Un prototype fonctionnel d'un système de pancréas artificiel en boucle fermée est développé en utilisant un microcontrôleur ESP32, une logique de contrôle basée sur des seuils et un actionnement précis par moteur pas à pas, démontrant avec succès les principes fondamentaux de l'administration automatisée d'insuline sur une plateforme modulaire et économique. Cette recherche comble efficacement le fossé entre l'analyse théorique de stabilité basée sur les LMI et les exigences pratiques de l'usage ambulatoire, ouvrant la voie à des systèmes d'administration d'insuline plus fiables, précis et personnalisés.

Mots-clés : Estimation d'état, systèmes non linéaires incertains, diabète de type 1, régulation glycémique, observateurs de Luenberger, théorie de la stabilité de Lyapunov, inégalités matricielles linéaires (LMI), modèle minimal de Bergman, pancréas artificiel, boucle fermée, microcontrôleur ESP32, moteur pas à pas, prototype fonctionnel.

Abbreviations and Acronyms

ADC	: Analog-to-Digital Converter
CGM	: Continuous Glucose Monitor
CPU	: Central Processing Unit
Feasp	: MATLAB-based convex optimization solver
EKF	: Extended Kalman Filter
EMI	: Electromagnetic Interference
ESP32	: Espressif Systems 32-bit Microcontroller
FDA	: U.S. Food and Drug Administration
GPIO	: General Purpose Input/Output
HIL	: Hardware-in-the-Loop
I2C	: Inter-Integrated Circuit
LMI	: Linear Matrix Inequality
LTI	: Linear Time-Invariant
MPC	: Model Predictive Control
NEMA	: National Electrical Manufacturers Association
OLED	: Organic Light Emitting Diode
PCB	: Printed Circuit Board
PID	: Proportional-Integral-Derivative
PMA	: Premarket Approval
PWM	: Pulse Width Modulation
SPD	: Symmetric Positive Definite
SPI	: Serial Peripheral Interface
SRAM	: Static Random-Access Memory
T1DM	: Type 1 Diabetes Mellitus
UART	: Universal Asynchronous Receiver-Transmitter

Mathematical Symbols

$x(t)$	: State vector
$\hat{x}(t)$	: Estimated state vector
$u(t)$	: Control input vector
$y(t)$	: Output vector
$e(t)$	: Estimation error vector
A, B, C, E	: System matrices
A_L, B_L, E_L	: Linear subsystem matrices
L	: Observer gain matrix
P	: Symmetric positive definite matrix (Lyapunov)
Q	: Positive definite matrix
R	: Change of variable: $R = PL$
$f(t)$	: Nonlinear compensation term
$v(t)$	: Generalized unknown input vector
$d(t)$	: External perturbation vector
d_L	: Linear subsystem disturbance
$d_{\max}$	: Maximum perturbation bound
$v_{\max}$	: Maximum unknown input bound
$x_{\max}$	: Maximum state bound
$u_{\max}$	: Maximum control input bound
α, β	: Bounds on nonlinear function $g(\cdot)$
a_1	: Non-zero scalar coupling parameter
x_{NL}	: Nonlinear part of state vector
x_L	: Linear part of state vector
$G(t)$	: Plasma glucose concentration
$X(t)$	: Insulin activity

$I(t)$: Plasma insulin concentration

G_b : Basal glucose level

I_b : Basal insulin level

p_1, p_2, p_3, p_4 : Physiological parameters

$\lambda_{\min}$: Minimum eigenvalue

$\lambda_{\max}$: Maximum eigenvalue

$\| \cdot \|$: Euclidean norm

$\mathcal{O}$: Observability matrix

$\text{rank}(\cdot)$: Rank of a matrix

$\tilde{y}(t)$: Output estimation error (residual)

b : Convergence radius of estimation error

$F(x)$: LMI expression

F_0, F_i : Symmetric matrices in LMI

x_i : Variables in LMI

$\mathbb{R}^m$: m-dimensional real vector space

m : Number of variables in LMI

$\mathbb{R}^{n \times n}$: n $\times$ n real matrix space

ϵ : Tuning parameter (Young's inequality)

$\dot{x}$: Time derivative of state vector

$\dot{\hat{x}}$: Time derivative of estimated state

$\dot{e}$: Time derivative of estimation error

$x_d(t)$: Deviation state vector

$x_{1_d}, x_{2_d}, x_{3_d}$: Deviation state components

x_0 : Physiological steady-state operating point

$D(t)$: Exogenous glucose appearance rate

X_I : Insulin action (dimensionless)

Contents

List of figures 16

General Introduction **17**

Chapter 1: Presentation of the Considered Class of Nonlinear Systems **21**

1.1 Introduction 22

1.2 Theoretical Foundations of State Estimation 22

1.2.1 Concept of Observability 23

1.2.2 Operating Principle of the Luenberger Observer 23

1.3 Presentation of the Class 25

1.4 Transformation into a Linear System with Unknown Input . . 26

1.4.1 Conclusion 28

Bibliography 29

Chapter 2: Observer Designs **30**

2.0.1 Introduction 31

2.1 Observer Design without Compensation of the Unknown Input : Method 1 31

2.2 System Framework 32

2.3 Proof of Convergence 33

2.4 Observer Design with Unknown Input Compensation: Method 2 35

2.4.1 Observer Structure 35

2.4.2 Real-Time Meal Disturbance Estimation 36

2.4.3 Proof of Convergence 37

2.5 Observer Design with Unknown Input Compensation and Estimation: Method 3 . . . 39
2.5.1 Observer Structure . . . 39
2.6 Proof of Convergence . . . 41
2.7 Illustrative Example . . . 42
2.7.1 Conclusion . . . 42
Bibliography . . . 43

Chapter 3: Application on an Artificial Pancreas System 44
3.0.1 Introduction . . . 45
3.1 Presentation of the Artificial Pancreas . . . 45
3.2 Modelisation of the artificial pancreas . . . 46
3.2.1 Physiological Parameter and State Envelope Profiles . 46
3.2.2 The Physical State Representation (x-Model) . . . 46
3.2.3 The Deviation State Representation (x_d-Model) . . . 48
3.3 Mapping the Deviation Model to the Considered Class . . . 49
3.3.1 State Vector Partitioning . . . 49
3.3.2 Coupling Parameter and Non-linear Function Extraction 50
3.3.3 Linear Sub-system Matrix Formulation . . . 50
3.4 Final representation: . . . 51
3.4.1 Conclusion . . . 52
Bibliography . . . 53

Chapter 4: Simulation and Prototype 54
4.0.1 Introduction . . . 55
4.1 Parameters and Configuration . . . 55
4.1.1 Analytical Formulation of the Meal Disturbance Signal 56
4.1.2 Numerical Implementation . . . 57
4.1.3 Graphical Representation and Axis Definition . . . 60
4.2 Comparative Performance Analysis of Proposed Robust Observers . . . 61
4.2.1 Performance Evaluation of Method 1 . . . 61

4.2.2 Performance Evaluation of Method 2 63
4.2.3 Comparative Discussion: Method 1 vs. Method 2 . . . 64
4.2.4 Performance Evaluation of Method 3 66
4.3 Prototype 68
4.4 System Aim and Overview 68
4.4.1 Project Objective 68
4.4.2 System Concept 69
4.5 System Components and Technical Specifications 70
4.5.1 Microcontroller: ESP32 DevKit V1 71
4.5.2 Glucose Sensor: Emulated CGM (Potentiometer) . . . 72
4.5.3 Human-Machine Interface: OLED SSD1306 74
4.5.4 Motor Driver: A4988 Microstepping Driver 75
4.5.5 Pump Actuator: Bipolar Stepper Motor (NEMA 17) . 76
4.6 System Wiring and Architecture 77
4.6.1 Logical Wiring Table 77
4.6.2 Wiring Diagram Interpretation 77
4.7 System Operation 78
4.7.1 Operational Logic 78
4.7.2 System Operation Cycle 78
4.7.3 Dosing Determination 79
4.8 Design Analysis and Performance Projections 79
4.8.1 Theoretical Performance Analysis 79
4.8.2 Dosing Precision Analysis 80
4.8.3 Power Consumption Analysis 80
4.8.4 System Stability Analysis 81
4.9 Comparative Analysis 81
4.10 Observations 81
4.10.1 Summary of Achievements 81
4.10.2 System Strengths 82
4.10.3 Limitations 82
4.10.4 Future Work 82

4.10.5 Conclusion 84
Bibliography 85
General Conclusion 89
Bibliography 91

List of Figures

1.1 Diagram of an oversimplified State Observer system. 24
4.1 Glucose, Insulin Action trajectories and their respective errors. 61
4.2 Plasmatic Insulin trajectory and its error. 61
4.3 Glucose, Insulin Action trajectories and their respective errors. 63
4.4 Plasmatic Insulin, Meal Disturbance Estimation and their errors. 63
4.5 Glucose, Insulin Action trajectories and their respective errors. 66
4.6 Plasmatic Insulin, Meal Disturbance Estimation and their respective errors. 66
4.7 System implementation 69
4.8 A simulated prototype 70
4.9 ESP32 DevKit V1 71
4.10 Potentiometer 72
4.11 OLED SSD1306 74
4.12 A4988 Driver used 75
4.13 Bipolar Stepper Motor 76

General Introduction

In the field of automatic control, the monitoring, optimization, and diagnosis of dynamic processes heavily rely on the real-time knowledge of all internal state variables. However, in physical applications, directly measuring the complete state vector is frequently impossible due to economic constraints, sensor technology limitations, or physical inaccessibility. To overcome this fundamental lack of information, control engineering utilizes state observers—software algorithms that exploit a system's mathematical model alongside available inputs and outputs to asymptotically reconstruct unmeasured internal states.

While state estimation is a mature framework for linear time-invariant systems, real-world systems are inherently nonlinear. In safety-critical applications—such as biomedical engineering and automated medical delivery devices—linear approximations are insufficient. This thesis focuses on the design of robust nonlinear observers, specifically applied to glycemic regulation via an artificial pancreas, establishing a rigorous framework that bridges advanced control theory with life-saving medical automation.

Context and Timeliness of the Research

The automated management of Type 1 Diabetes Mellitus (T1DM) has emerged as a critical, highly active research area at the intersection of control theory and biomedical science. Traditional clinical methods of insulin administration often fail to maintain normoglycemia due to severe external disturbances, such as unannounced meal intake, metabolic variations, and

physical activity. Consequently, designing a reliable "artificial pancreas" is of paramount importance today. It directly impacts patient safety, minimizes long-term diabetic complications, and significantly improves the quality of life by eliminating the burden of manual insulin calculations.

An artificial pancreas operates as a closed-loop system consisting of a continuous glucose monitor (CGM), a control algorithm, and an insulin pump. A critical challenge within this architecture is that key metabolic states—such as plasma insulin concentration or the rate of gastric emptying—cannot be directly measured in vivo. Robust state estimation is therefore indispensable for reconstructing these hidden variables in real time, providing the control loop with accurate data.

The Research Problematic

The human glucose-insulin regulatory system is highly nonlinear, time-varying, and subject to intense inter- and intra-patient uncertainties. Classic nonlinear estimation techniques, such as the Extended Kalman Filter (EKF), rely on local Taylor approximations and lack global stability guarantees, making them risky for medical deployment. High-gain observers can handle nonlinearities but are highly sensitive to measurement noise.

Therefore, the central problematic addressed in this thesis is: *How can we synthesize a state estimation observer that guarantees robust, global asymptotic stability for highly nonlinear systems while simultaneously rejecting parametric uncertainties, measurement noise, and external meal disturbances?*

The core hypothesis of this work is that by leveraging Lyapunov stability theory and reformulating estimation constraints into convex Linear Matrix Inequalities (LMIs), we can compute optimal observer gains that guarantee safe, ultimate boundedness of estimation errors under realistic clinical conditions.

Adopted Methodology

To resolve this problematic, the methodology adopted in this work combines theoretical synthesis, numerical optimization, and experimental hardware validation. The research is executed through the following structured phases:

- **Mathematical Modeling:** Utilizing nonlinear state-space formulations to accurately capture the multi-compartmental metabolic dynamics of glucose-insulin interactions based on the Bergman Minimal Model.
- **Theoretical Observer Synthesis:** Applying Lyapunov stability functions to derive robust estimation criteria, which are mathematically transformed into Linear Matrix Inequalities (LMIs) to ensure global convergence.
- **Convex Optimization:** Utilizing advanced numerical solvers (specifically the `feasp` solver within the MATLAB environment) to efficiently solve the LMI design constraints and obtain the observer gain matrices.
- **Practical Verification:** Simulating the closed-loop artificial pancreas under severe meal disturbance profiles in MATLAB, followed by deploying the control and automation principles onto a physical ESP32-based embedded microcontroller prototype to validate technical feasibility.

Thesis Outline

To systematically present our findings and align with the actual research structure, this manuscript is organized into four distinct chapters: **Chapter 1: Presentation of the Considered Class of Nonlinear Systems** establishes the theoretical baseline of state estimation, defining classical observability criteria and Luenberger observer operations before formalizing the specific class of triangular nonlinear systems used throughout this study.

Chapter 2: Observer Designs details the mathematical synthesis and convergence proofs for three progressive estimation frameworks: a baseline

observer without unknown input compensation (Method 1), an observer incorporating explicit unknown input compensation (Method 2), and an aggressive observer integrating unknown input compensation with dynamic estimation (Method 3).

Chapter 3: Application to the Artificial Pancreas System handles the physiological mapping of the biomedical problem. It transforms the three-dimensional Bergman Minimal Model into deviation coordinates and partitions the state variables to fully fit the established triangular robust system class.

Chapter 4: Simulation and Prototype presents a comparative numerical performance analysis conducted in MATLAB. By tracking plasma glucose trajectories, insulin action, and plasmatic insulin peaks under meal disturbances, it demonstrates the definitive advantages and accuracy of Method 3.inear Luenberger obse

Chapter 1:

Presentation of the Considered Class of Nonlinear Systems

1.1 Introduction

In the field of automatic control, the control, monitoring, and diagnosis of dynamic processes generally require real-time knowledge of all the system's state variables. However, in practice, it is often difficult or impossible to directly measure the entirety of this state vector due to economic, technological, or physical accessibility constraints. To overcome this lack of information, state observers (or estimators) are utilized. An observer is a software algorithm that, by exploiting the mathematical model of the system along with the only available measurements (inputs and outputs), asymptotically reconstructs the unmeasured internal state [1].

The objective of this chapter is to first establish the classical foundations of state estimation and observability criteria. Following this theoretical baseline, we formalize the mathematical structure of the specific class of nonlinear systems on which our observer synthesis study will be focused. The significance of defining this class lies in the ability to exploit its structural properties—namely, a triangular decomposition combining a nonlinear dynamics with a linear substructure—in order to design robust estimation laws capable of handling model uncertainties and external perturbations [3].

1.2 Theoretical Foundations of State Estimation

Before addressing the complexity of non-linear structures, it is fundamental to formalize the study framework for linear time-invariant (LTI) systems. A continuous linear system is modeled in state-space form by the following classical system of equations [2]:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1.1}$$

Where $x(t) \in \mathbb{R}^n$ is the state vector representing the internal memory of the system, $u(t) \in \mathbb{R}^m$ is the input vector (control signals), $y(t) \in \mathbb{R}^p$ is

the output vector (sensor measurements), $A \in \mathbb{R}^{n \times n}$ is the system dynamic matrix, $B \in \mathbb{R}^{n \times m}$ is the input matrix, and $C \in \mathbb{R}^{p \times n}$ is the output matrix.

1.2.1 Concept of Observability

Observability is an intrinsic structural property of a system. It determines whether it is mathematically possible to reconstruct the state vector from the knowledge of the outputs and inputs over a finite time interval. A system is said to be fully observable if, for any unknown initial state $x(0)$, it is possible to uniquely determine this vector $x(0)$ from the observation of the output $y(t)$ and the control input $u(t)$ over a finite time interval $[0, T]$.

For an LTI system, observability can be verified algebraically using the Kalman observability matrix [2], denoted by $\mathcal{O}$, defined by:

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \in \mathbb{R}^{(n \cdot p) \times n} \tag{1.2}$$

According to Kalman's Theorem, the system is completely observable if and only if the matrix $\mathcal{O}$ has full column rank, that is: $\text{rank}(\mathcal{O}) = n$ [2]. If the rank is less than n, there exist hidden modes or states that do not influence the output $y(t)$, making the reconstruction of these states impossible with a conventional observer.

1.2.2 Operating Principle of the Luenberger Observer

A state observer does not merely simulate the mathematical model in parallel, which would quickly diverge due to modeling errors and unknown initial conditions. Instead, it integrates a closed-loop correction mechanism based on the discrepancy between the actual process output and the estimated output. Proposed by David Luenberger in the 1960s, the linear observer con-

stitutes the fundamental structure for state estimation in noise-free linear systems [1]. The observer replicates the structure of the system but injects a correction term proportional to the output error $\tilde{y}(t) = y(t) - \hat{y}(t)$, often referred to as the residual or innovation. The structural architecture of this estimation loop is shown in Figure 1.1.

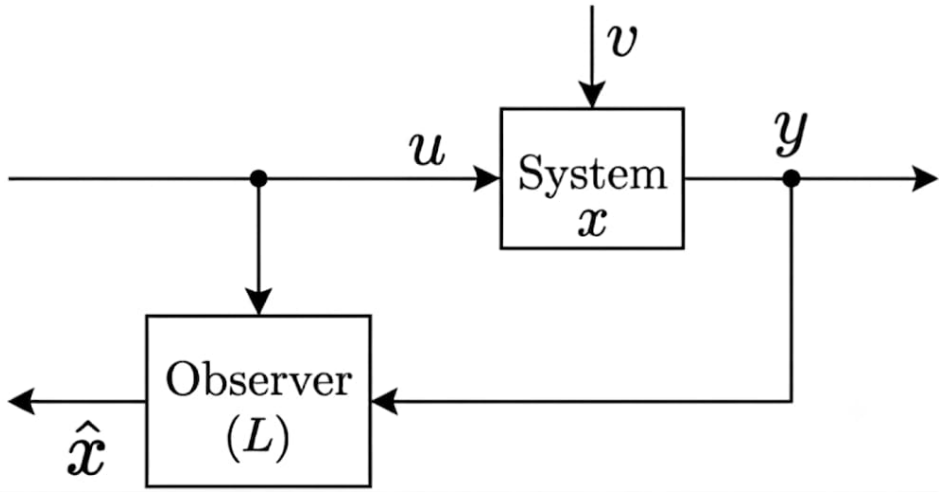

Figure 1.1: Diagram of an oversimplified State Observer system.

The dynamic equation of the linear Luenberger observer [1, 5] is formulated as:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) - L(C\hat{x}(t) - y(t)) \tag{1.3}$$

Where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state and $L \in \mathbb{R}^{n \times p}$ is the observer gain matrix, which must be synthesized to ensure convergence. To evaluate the estimation error dynamics, we define the error vector as $e(t) = \hat{x}(t) - x(t)$. By differentiating this error with respect to time in a nominal framework free of perturbations and unknown inputs, we obtain the nominal error dynamics [1, 5]:

$$\begin{aligned}
\dot{e}(t) &= \dot{\hat{x}}(t) - \dot{x}(t) \\
\dot{e}(t) &= \big(A\hat{x}(t) + Bu(t) - L(C\hat{x}(t) - y(t))\big) - \big(Ax(t) + Bu(t)\big) \\
\dot{e}(t) &= A(\hat{x}(t) - x(t)) - LC(\hat{x}(t) - x(t)) \\
\dot{e}(t) &= (A - LC)e(t)
\end{aligned} \tag{1.4}$$

The evolution of the error is therefore governed by a linear autonomous system whose transition matrix is $(A - LC)$. For the estimated state to converge toward the true state ($\lim_{t\to\infty} e(t) = 0$), the matrix $(A - LC)$ must be Hurwitz (all its eigenvalues must have strictly negative real parts). If the system is completely observable, the pole placement theorem guarantees that the gain L can be chosen to assign the eigenvalues of $(A - LC)$ arbitrarily, thereby regulating the convergence speed of the observer.

1.3 Presentation of the Class

Let the state vector $x = (x_{NL} \quad x_L)^T \in \mathbb{R}^n$ be composed of a nonlinear part $x_{NL} \in \mathbb{R}$ and a linear part $x_L = (x_{L,1}, \ldots, x_{L,n-1})^T \in \mathbb{R}^{n-1}$. Let us consider the following class of nonlinear systems [3], composed of a linear part and a nonlinear part:

$$\begin{cases} \dot{x}_{NL} = a_1 x_{L,1} + g(x_{NL}, x_L, t) x_{NL} + d_1 \\ \dot{x}_L = A_L x_L + B_L u + E_L d_L \end{cases} \tag{1.5}$$

with the output given by [3]:

$$y = x_{NL} \tag{1.6}$$

Where $a_1 \neq 0$ is a non-zero scalar, the state $x \in E \subset \mathbb{R}^n$, and the function $\alpha \leq g(x_{NL}, x_{L,1}, t) \leq \beta$ is a known function depending on the state and/or time, bounded by the constants α and β. The perturbation $d(t) = (d_1(t) \quad d_L(t))^T$ is an unknown bounded external perturbation [3], such that:

$$\|d(t)\| \leq d_{max} \tag{1.7}$$

Where d_{max} is a known constant. The matrices A_L, B_L, and E_L are known constant matrices of appropriate dimensions. Assume that system (1.5) satisfies the following assumptions:

Assumptions

(H1.) The state $x \in E \subset \mathbb{R}^n$ lies within a bounded domain with $\|x\| \leq x_{max}$, where x_{max} is known [3].

(H2.) The perturbation vector $d(t)$ is unknown but bounded with a known bound $\|d(t)\| \leq d_{max}$.

(H3.) The control input $u \in U \subset \mathbb{R}$ is bounded with $\|u\| \leq u_{max}$, where u_{max} is known.

Example

Consider the following system [3]:

$$\begin{cases} \dot{x}_1 = x_2 + \cos(x_2)x_1 + d \\ \dot{x}_2 = -x_3 \\ \dot{x}_3 = u \\ y = x_1 \end{cases} \tag{1.8}$$

It is clear that system (1.8) matches the general form with $x_{NL} = x_1$, $x_L = (x_2 \quad x_3)^T$, $g(x_{NL}, x_L, t) = \cos(x_2)$, and the following structural matrices [3]:

$$A_L = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad B_L = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_L = 0 \tag{1.9}$$

1.4 Transformation into a Linear System with Unknown Input

To facilitate the synthesis of the state observer, the considered system class (1.5) can be reformulated into a compact linear time-invariant (LTI) form affected by an unknown input vector [4]. Let us define the auxiliary variable $v_1(t)$ to encapsulate the system's nonlinearities and external disturbances:

$$v_1(t) = g(x_{NL}, x_L, t)x_{NL} + d_1 \tag{1.10}$$

Using this definition, the dynamic equations of the system class can be rewritten as follows [4]:

$$\begin{cases} \dot{x}_{NL} = a_1 x_{L,1} + v_1 \\ \dot{x}_L = A_L x_L + B_L u + E_L d_L \end{cases} \tag{1.11}$$

with output [4]:

$$y = Cx \tag{1.12}$$

Where the output matrix is defined as $C = [1 \quad 0 \quad 0 \quad \dots \quad 0]$ of dimension $(1 \times n)$. In a more compact state-space representation, the overall coupled system can be written in the following generalized form [4]:

$$\begin{cases} \dot{x} = Ax + Bu + Ev \\ y = Cx \end{cases} \tag{1.13}$$

Where the augmented state vector $x \in \mathbb{R}^n$ and the generalized unknown input vector v are defined as [4]:

$$x = [x_1, x_2, \dots, x_{n-1}, x_n]^T = [x_{NL} \quad x_L^T]^T, \quad v = \begin{bmatrix} v_1 \\ d_L \end{bmatrix} \tag{1.14}$$

The structural, control, and disturbance matrices of this compact representation are given by [4]:

$$A = \begin{bmatrix} a_1 & 0 \\ A_L & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ B_L \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & E_L \end{bmatrix} \tag{1.15}$$

Example:

To demonstrate, let us reconsider the previous academic example [4]:

$$\begin{cases} \dot{x}_1 = x_2 + \cos(x_2)x_1 + d \\ \dot{x}_2 = -x_3 \\ \dot{x}_3 = u \\ y = x_1 \end{cases} \tag{1.16}$$

By defining the unknown input variable as $v(t) = \cos(x_2)x_1 + d(t)$, the system equations (1.16) can be straightforwardly mapped into the compact state-space form (1.13) [4]:

$$\begin{cases} \dot{x} = Ax + Bu + Ev \\ y = Cx \end{cases} \tag{1.17}$$

With the following constant system matrices [4]:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{1.18}$$

1.4.1 Conclusion

In this chapter, classical fundamentals of state estimation and algebraic observability criteria were established for linear time-invariant systems. Through this, we formalized the specific class of triangular nonlinear systems forming the core framework of this research. By introducing an auxiliary variable to isolate state-dependent nonlinearities and external disturbances, we demonstrated how this complex coupled class systematically transforms into a compact linear system with an unknown input vector. This new structure provides the framework necessary for both the robust observer synthesis and stability analysis detailed in the next chapter.

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Chapter 2:

Observer Designs

2.0.1 Introduction

Building upon the mathematical framework of the triangular system class established in the previous chapter, this chapter details the core theoretical contributions of this thesis: the synthesis and convergence analysis of three distinct robust state observers. To systematically address the challenges of external disturbances and model nonlinearities, three progressive methodologies are developed, adapted from the framework of [3]. We begin with a baseline observer operating without explicit unknown input compensation, establish an active nonlinear cancellation framework to drive estimation errors to zero, and ultimately formulate an aggressive observer utilizing state-dependent constraints, with significant modifications and extensions to the original approaches in [3]. Each methodology is derived using Lyapunov stability theory, and the design constraints are mapped into convex Linear Matrix Inequalities (LMIs) to provide strict, global convergence proofs [5, 3].

2.1 Observer Design without Compensation of the Unknown Input : Method 1

First, let us recall that the equations of the considered system are of the form presented in the compact matrix framework. In this section, we consider the same observer as the one used in the case of linear systems without unknown input, without introducing any specific mechanism for compensating the unknown input $v(t)$ [1].

Despite the absence of this compensation, the observer remains applicable and allows for a satisfactory estimation of the system states. However, its performance and the convergence of the estimation error depend on the amplitude of the unknown input, as established in the general framework of [3]. More precisely, the proper functioning of the observer is guaranteed under the hypothesis that $v(t)$ remains bounded and that its maximum norm does not exceed a limit value determined by the stability conditions of the estimation system [2, 3].

2.2 System Framework

Consider the previous class of systems represented in the following compact state-space form [3]:

$$\begin{cases} \dot{x} = Ax + Bu + Ev \\ y = Cx \end{cases} \tag{2.1}$$

We propose the following state observer structure [1]:

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y) \tag{2.2}$$

Theorem 1. The estimated state $\hat{x}$ of the observer (2.2) converges asymptotically toward the true state x of system (2.1) if the observer gain L is computed such that the following Linear Matrix Inequality (LMI) is satisfied. That is, there must exist a symmetric positive definite (SPD) matrix $P > 0$ such that [5]:

$$-Q = P(A - LC) + (A - LC)^T P < 0 \tag{2.3}$$

where $Q > 0$, and provided that the bound on the perturbation vector satisfies $\|v(t)\| \leq v_{max}$ under the following condition [3]:

$$v_{max} \leq \frac{\lambda_{min}(Q)}{2\|E\|\lambda_{max}(P)} b \tag{2.4}$$

where $\lambda_{min}(Q)$ is the minimum eigenvalue of Q, $\lambda_{max}(P)$ is the maximum eigenvalue of P, and b is the convergence radius of the estimation error $e = \hat{x} - x$, such that $\|e\| \leq b$.

2.3 Proof of Convergence

The dynamics of the state estimation error $e = \hat{x} - x$ are given by subtracting the system equations from the observer equations [3, 7, 8, 9]:

$$\dot{e} = (A - LC)e - Ev \tag{2.5}$$

To solve the LMI (2.3), we apply the change of variables $R = PL$, where R is an arbitrary matrix of appropriate dimensions [5]. This yields the standard LMI formulation to be solved for $P > 0$ and R [5]:

$$PA + A^T P - RC - C^T R^T < 0 \tag{2.6}$$

We note that $R = PL \implies L = P^{-1}R$. Let us define the candidate Lyapunov function as [2]:

$$V = e^T P e \tag{2.7}$$

Differentiating V with respect to time leads to [2]:

$$\dot{V} = 2e^T P \dot{e} \tag{2.8}$$

Substituting the error dynamics equation (2.5) into the derivative yields [2]:

$$\begin{aligned} \dot{V} &= 2e^T P\big((A - LC)e - Ev\big) \\ &= 2e^T P(A - LC)e - 2e^T PEv \\ &= e^T\big(P(A - LC) + (A - LC)^T P\big)e - 2e^T PEv \end{aligned} \tag{2.9}$$

Substituting the design matrix equality $-Q = P(A-LC)+(A-LC)^T P < 0$ with $Q > 0$, we obtain [2]:

$$\dot{V} = -e^T Q e - 2e^T PEv \tag{2.10}$$

Applying norm inequalities [2], this can be upper-bounded as [2]:

$$\dot{V} \leq -e^T Q e + 2\|e\|\|P\|\|E\|\|v\| \tag{2.11}$$

Using the properties of eigenvalues and the maximum bound of the perturbation vector v_{max}, it follows that [2]:

$$\dot{V} \leq -\lambda_{min}(Q)\|e\|^2 + 2\lambda_{max}(P)\|E\|\|e\|v_{max} \tag{2.12}$$

Factoring out the error norm $\|e\|$ results in [3]:

$$\dot{V} \leq -\big(\lambda_{min}(Q)\|e\| - 2\lambda_{max}(P)\|E\|v_{max}\big)\|e\| \tag{2.13}$$

Consequently, to guarantee that the Lyapunov derivative remains strictly negative ($\dot{V} \leq 0$), the following condition must hold [2]:

$$\lambda_{min}(Q)\|e\| - 2\lambda_{max}(P)\|E\|v_{max} \geq 0 \tag{2.14}$$

This condition ensures ultimate boundedness in finite time and can be explicitly rewritten as [3]:

$$v_{max} \leq \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)\|E\|}\|e\| \tag{2.15}$$

Assuming that the estimation error ultimately converges inside a ball of radius b such that $\|e\| \leq b$, we achieve the final required bound [3]:

$$v_{max} \leq \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)\|E\|}b \tag{2.16}$$

This completes the proof.

2.4 Observer Design with Unknown Input Compensation: Method 2

In this section, we extend the estimation approach to include an active compensation mechanism for the unknown input $v(t)$, inspired by the framework of [3]. While the previous method relied on the robustness of the observer to handle perturbations within a bounded range, we now introduce a nonlinear compensation term $f(t)$ designed to actively mitigate the influence of the unknown input on the estimation error dynamics, with a novel formulation adapted from [3]. By integrating this compensation term, we aim to achieve asymptotic convergence of the estimation error to zero, effectively counteracting the effects of the disturbance $v(t)$ under the assumption that its bound v_{max} is known [2, 3].

2.4.1 Observer Structure

We consider the same class of linear systems as defined previously. We propose the following augmented state observer structure incorporating the nonlinear compensation term $f(t)$ [3, 7, 8, 9]:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + f(t) - L(C\hat{x}(t) - y(t)) \tag{2.17}$$

In this work we propose an extension of the robust estimation framework presented in [3]. To effectively attenuate the effects of the system nonlinearities and ensure the asymptotic convergence of the estimation error, a new nonlinear compensation law $f(t)$ is designed as follows [3]:

$$f(t) = -\frac{\|P\|^2\|E\|^2 v_{\max}^2}{\lambda_{\min}(Q)} P^{-1}C^T \frac{Ce}{\|Ce\|^2} \tag{2.18}$$

By exploiting the structural properties specific to the considered artificial pancreas model, we can establish that $E = C^T$, which directly implies $\|E\| = \|C\| = 1$. Furthermore, noting that $\|P\| = \lambda_{\max}(P)$, the proposed

compensation law simplifies to [3]:

$$f(t) = -\frac{(\lambda_{\max}(P))^2 \|v_{\max}\|^2}{\lambda_{\min}(Q)} \frac{P^{-1}C^T Ce}{\|Ce\|^2} \tag{2.19}$$

From an implementation perspective, a direct evaluation of (2.19) can lead to numerical instability or singularity issues when the output estimation error approaches zero. To overcome this practical limitation and avoid division by zero, the control law is implemented using a regularization parameter ε as follows [3, 9]:

$$f(t) = -\nu \frac{P^{-1}C^T e_1}{|e_1|^2 + \varepsilon} \tag{2.20}$$

where the constant tuning gain ν and the regulated error vector e_1 are defined by [3]:

$$\nu = \frac{(\lambda_{\max}(P))^2 \|v_{\max}\|^2}{\lambda_{\min}(Q)}, \quad e_1 = Ce \tag{2.21}$$

and $\varepsilon > 0$ is a small strictly positive scalar chosen to ensure smooth numerical behavior in the vicinity of the equilibrium.

2.4.2 Real-Time Meal Disturbance Estimation

A crucial advantage of the proposed robust observer framework is its ability to actively reconstruct the unmeasured exogenous disturbances affecting the metabolic system. While the nonlinear compensation term $f(t)$ directly handles the lumped unknown input vector $v(t)$, it encapsulates both external inputs and internal nonlinear dynamics. Specifically, the total lumped disturbance is defined as $v(t) = -x_1 x_2 + d(t)$.

To isolate the actual rate of glucose appearance from meal intake, the estimated meal disturbance signal $\hat{d}(t)$ is reconstructed by decoupling the estimated state nonlinearities from the compensation law as follows [3]:

$$\begin{aligned} \hat{d}(t) &= Cf(t) + \hat{x}_1(t)\hat{x}_2(t) \\ \hat{v}(t) &= Cf(t) \end{aligned} \tag{2.22}$$

where $\hat{x}_1(t)$ and $\hat{x}_2(t)$ represent the real-time observer estimates of the plasma glucose and the remote compartment insulin activity, respectively. By utilizing (2.22), the artificial pancreas system can successfully synthesize a dynamic estimate of the patient's caloric intake without relying on subjective, manual meal logging.

Theorem 2. The estimated state $\hat{x}$ of the observer (2.17) converges to the true state x of the system if the gain matrix L is computed such that there exists a symmetric positive definite matrix $P > 0$ satisfying the LMI [5]:

$$PA + A^T P - RC - C^T R^T < 0 \tag{2.23}$$

2.4.3 Proof of Convergence

The state observation error is defined as $e = \hat{x} - x$. Its derivative yields the following dynamic expression [3]:

$$\dot{e} = (A - LC)e + f - Ev \tag{2.24}$$

We define the candidate Lyapunov function as $V = e^T Pe$, with its time derivative given by [2, 7, 8, 9]:

$$\begin{aligned} \dot{V} &= 2e^T P\dot{e} \\ &= 2e^T P(A - LC)e + 2e^T Pf - 2e^T PEv \\ &= e^T \left[P(A - LC) + (A - LC)^T P\right] e + 2e^T Pf - 2e^T PEv \end{aligned} \tag{2.25}$$

Choosing L such that $P(A - LC) + (A - LC)^T P = -Q$ for an SPD matrix $Q > 0$, we have [2]:

$$\dot{V} = -e^T Qe + 2e^T Pf - 2e^T PEv \tag{2.26}$$

Using standard eigenvalue bounds, where $\lambda_{\min}(Q)\|e\|^2 \leq e^T Qe \leq \lambda_{\max}(Q)\|e\|^2$,

splitting the quadratic term yields [2]:

$$\dot{V} \leq -\frac{\lambda_{\min}(Q)}{2}\|e\|^2 - \frac{\lambda_{\min}(Q)}{2}\|e\|^2 + 2e^T Pf - 2e^T PEv \tag{2.27}$$

To guarantee asymptotic stability such that $\dot{V} \leq -\frac{\lambda_{\min}(Q)}{2}\|e\|^2$, we must satisfy the following inequality condition [2]:

$$-\frac{\lambda_{\min}(Q)}{2}\|e\|^2 + 2e^T Pf - 2e^T PEv \leq 0 \tag{2.28}$$

By invoking Young's inequality [2] on the perturbation term [2]:

$$-2e^T PEv \leq 2\|P\|\|E\|\|e\|\|v\| \leq \epsilon\|P\|\|E\|\|e\|^2 + \frac{1}{\epsilon}\|P\|\|E\|\|v\|^2 \tag{2.29}$$

Setting the tuning parameter to $\epsilon = \frac{\lambda_{\min}(Q)}{2\|P\|\|E\|}$, the condition simplifies to [2]:

$$-\frac{\lambda_{\min}(Q)}{2}\|e\|^2 + 2e^T Pf + \frac{\lambda_{\min}(Q)}{2}\|e\|^2 + \frac{2}{\lambda_{\min}(Q)}\|P\|^2\|E\|^2\|v\|^2 \leq 0 \tag{2.30}$$

which further reduces to [3]:

$$2e^T Pf + \frac{2}{\lambda_{\min}(Q)}\|P\|^2\|E\|^2 v_{\max}^2 \leq 0 \tag{2.31}$$

Substituting the selected definition of $f(t)$ from (2.18) [3]:

$$2e^T P\left(-\frac{\|P\|^2\|E\|^2 v_{\max}^2}{\lambda_{\min}(Q)} P^{-1}C^T \frac{Ce}{\|Ce\|^2}\right) + \frac{2}{\lambda_{\min}(Q)}\|P\|^2\|E\|^2 v_{\max}^2 \leq 0 \tag{2.32}$$

$$-\frac{2\|P\|^2\|E\|^2 v_{\max}^2}{\lambda_{\min}(Q)}\frac{(Ce)^T(Ce)}{\|Ce\|^2} + \frac{2}{\lambda_{\min}(Q)}\|P\|^2\|E\|^2 v_{\max}^2 \leq 0 \tag{2.33}$$

$$-\frac{2}{\lambda_{\min}(Q)}\|P\|^2\|E\|^2 v_{\max}^2 + \frac{2}{\lambda_{\min}(Q)}\|P\|^2\|E\|^2 v_{\max}^2 \leq 0 \tag{2.34}$$

The inequality condition is identically satisfied.

2.5 Observer Design with Unknown Input Compensation and Estimation: Method 3

An alternative approach to state reconstruction utilizes state-dependent constraints to dominate modeling uncertainties, adapted from the framework of [3]. In this method, we refine the compensation law to explicitly incorporate the bound on the system state, providing a robust mechanism for handling unknown inputs, with a novel formulation that extends the approach in [3]. This approach ensures that the estimation error is driven to zero even when the perturbation $v(t)$ is significant, provided the system state remains within a predefined bounded region [2, 3].

2.5.1 Observer Structure

Consider the following alternate observer structure [3]:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + f(t) - L(C\hat{x}(t) - y(t)) \tag{2.35}$$

In this work, the nonlinear compensation term $f(t)$ is designed as a novel extension of the approach in [3], assuming the state and disturbance vectors are bounded such that $\|x(t)\| \leq x_{\max}$ and $\|v(t)\| \leq v_{\max}$ [3]:

$$f(t) = -\|P\|\|E\|v_{\max}\big(\|\hat{x}(t)\| + x_{\max}\big)P^{-1}C^T\frac{Ce}{\|Ce\|^2} \tag{2.36}$$

By exploiting the structural properties specific to the considered artificial pancreas model, we can establish that $E = C^T$, which directly implies $\|E\| = \|C\| = 1$. Furthermore, noting that $\|P\| = \lambda_{\max}(P)$, the proposed

compensation law simplifies to [3]:

$$f(t) = -\lambda_{\max}(P) v_{\max} \big(\|\hat{x}(t)\| + x_{\max} \big) \frac{P^{-1} C^T C e}{\|Ce\|^2} \tag{2.37}$$

From an implementation perspective, a direct evaluation of (2.37) can lead to numerical instability or singularity issues when the output estimation error approaches zero. To overcome this practical limitation and avoid division by zero, the control law is implemented using a regularization parameter ε as follows [3, 9]:

$$f(t) = -\nu \big(\|\hat{x}(t)\| + x_{\max} \big) \frac{P^{-1} C^T e_1}{|e_1|^2 + \varepsilon} \tag{2.38}$$

where the constant tuning gain ν and the regulated error vector e_1 are defined by [3]:

$$\nu = \lambda_{\max}(P) v_{\max}, \quad e_1 = Ce \tag{2.39}$$

and $\varepsilon > 0$ is a small strictly positive scalar chosen to ensure smooth numerical behavior in the vicinity of the equilibrium.

Meal disturbance estimation [3]:

$$\begin{aligned} \hat{d}(t) &= C f(t) + \hat{x}_1(t) \hat{x}_2(t) \\ \hat{v}(t) &= C f(t) \end{aligned} \tag{2.40}$$

Theorem 3. [5] The estimated state $\hat{x}$ of the observer (2.35) converges asymptotically to the true state vector x if the gain matrix L is computed through an SPD matrix $P > 0$ satisfying the LMI condition [5]:

$$PA + A^T P - RC - C^T R^T < 0 \tag{2.41}$$

2.6 Proof of Convergence

Given the observation error definition $e = \hat{x} - x$, the error dynamic response is formulated as [3]:

$$\dot{e} = (A - LC)e + f - Ev \tag{2.42}$$

Defining the candidate Lyapunov function as $V = e^T Pe$, its derivative is expressed as [2]:

$$\begin{aligned} \dot{V} &= 2e^T P\dot{e} \\ &= 2e^T P(A - LC)e + 2e^T Pf - 2e^T PEv \\ &= e^T\big(P(A - LC) + (A - LC)^T P\big)e + 2e^T Pf - 2e^T PEv \end{aligned} \tag{2.43}$$

By defining $P(A - LC) + (A - LC)^T P = -Q$ with $Q > 0$, the derivative yields [2]:

$$\dot{V} = -e^T Qe + 2e^T Pf - 2e^T PEv \tag{2.44}$$

Using the properties of the quadratic form [2], we write [2]:

$$\dot{V} \leq -\lambda_{min}(Q)\|e\|^2 + 2e^T Pf - 2e^T PEv \tag{2.45}$$

To achieve definitive asymptotic convergence, the remaining terms must satisfy [2]:

$$2e^T Pf - 2e^T PEv \leq 0 \tag{2.46}$$

Taking the norm of the perturbation block, we have $-2e^T PEv \leq 2\|P\|\|E\|\|e\|\|v\|$ [2]. Since the absolute system state is bounded under the uniform assumptions ($\|x\| \leq x_{max}$), the triangle inequality on the error vector gives $\|e\| \leq \|\hat{x}(t)\| + x_{max}$ [2, 3]. Substituting this bound into the stability inequality and applying the structural definition of $f(t)$ from (2.36) confirms that the condition is met, ensuring asymptotic convergence [3, 2]. This represents a novel contribution that extends the standard framework by incorporating

state-dependent bounding constraints.

2.7 Illustrative Example

Returning to our previous system with $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, and $E = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, we apply the compensation law (2.36). By utilizing the bound x_{max}—the maximum expected value of the system states—the observer dynamically adjusts the gain $f(t)$ to cancel the disturbance $v(t) = \sin(t)$, following the approach adapted from [3] [3]:

$$f(t) = -\nu(\|\hat{x}(t)\| + x_{\max})P^{-1}C^T \frac{e_1}{|e_1|^2 + \varepsilon} \tag{2.47}$$

2.7.1 Conclusion

This chapter successfully detailed the mathematical design and formal convergence validation for three progressive estimation frameworks. Through a systematic application of Lyapunov stability theory [2] and Young's inequality, design constraints were successfully cast as convex Linear Matrix Inequalities (LMIs) solvable via standard computational toolboxes [5]. While Method 1 offers baseline robustness under bounded conditions, following the standard approach in [3], Methods 2 and 3 actively incorporate nonlinear compensation terms $f(t)$ to completely dominate unknown inputs, with significant adaptations and extensions to the original framework in [3]. Method 3, by dynamically scaling the compensation envelope using state estimates, yields the most aggressive theoretical error suppression, representing the primary novel contribution of this chapter. In the next chapter, these three synthesis paradigms will be mapped onto the physiological equations of the human glucose-insulin regulatory system [4, 6].

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Chapter 3:

Application on an Artificial Pancreas System

3.0.1 Introduction

Following the mathematical synthesis and formal stability proofs of the three robust observation frameworks developed in Chapter 2, this chapter addresses the concrete execution of these methods within a biomedical control engineering framework. The primary objective here is to transition from abstract algorithmic constraints to a structured physiological application. To achieve this, we detail the complete state-space mapping of glucose-insulin metabolic pathways, **adapted from** the Bergman Minimal Model framework [3]. We introduce the non-equilibrium mathematical variations of the system, establish the transformation of physiological metrics into standardized deviation coordinates, and systematically partition the resulting multi-compartmental model. This rigorous structural conditioning is what allows the complex biological dynamics to precisely fit the canonical triangular nonlinear system matrices required by our synthesized observers, following the methodology presented in [2] and [1].

3.1 Presentation of the Artificial Pancreas

The artificial pancreas is defined as a closed-loop glycemic regulation system aimed at restoring, in an automated manner, the deficient endocrine function in patients with type 1 diabetes [4, 7]. This biomedical system is composed of two distinct but complementary technological entities:

- **A Continuous Glucose Monitor (CGM):** which provides, in real-time, an estimate of the patient's interstitial glucose level.
- **An External Insulin Pump:** which ensures the administration of the hormonal dosage calculated by the control algorithm.

The central challenge of this closed-loop configuration lies in inter-individual variability and the unpredictability of exogenous inputs, particularly during meal ingestion [6]. Within the scope of this thesis, we model this carbohydrate ingestion as an unknown external perturbation, denoted as $d(t)$, for

which we assume a known upper bound, d_{max} [1]. This perturbation represents the major technical obstacle that our robust observer must overcome: maintaining a precise estimation of the glycemic state (x_1) and insulin action (x_2) despite the disturbed and uncertain nature of the metabolic dynamics.

The objective of this presentation is to lay the clinical and operational groundwork for the detailed physiological modeling that follows, where we will demonstrate that glycemic dynamics can be mathematically reformulated according to the robust triangular structure established in Chapter 1 [2, 1].

3.2 Modelisation of the artificial pancreas

3.2.1 Physiological Parameter and State Envelope Profiles

To define a realistic framework for state estimation and subsequent numerical validation, the physiological values, units, and system constraints extracted from patient datasets are introduced [2, 4]. Table 3.1 details the parameters alongside the operating ranges of the state variables for an adult patient profile. In order to construct a highly realistic evaluation scenario, the model parameters are treated as uncertain variables randomly chosen from specified ranges provided in Table 3.2. Notably, the parameter p_1—representing the endogenous insulin-independent glucose clearance rate—is physiologically negligible in Type 1 Diabetes Mellitus (T1DM) patients due to complete pancreatic beta-cell degradation.

3.2.2 The Physical State Representation (x-Model)

The metabolic system of a T1DM patient is modeled using a three-dimensional continuous-time state-space representation derived from the Bergman Minimal Model [3]. The state vector $x(t) \in \mathbb{R}^3$ describes the direct biological

Table 3.1: Nominal and range of parameters and state variables for T1DM patients. [8, 9, 10]

Parameters	**Nominal value (unit)**	**Range**
p_1	0 (min^{-1})	-
p_2	0.015 (min^{-1})	[0.01, 0.021]
p_3	2×10^{-6} (mU/l/min^2)	$[1 \times 10^{-6}, 3 \times 10^{-6}]$
p_4	0.2 (min^{-1})	[0.12, 0.3]
G_b	80 (mg/dl)	-
I_b	7 (mU/l)	-
States, $x(t)$	**Unit**	**Range**
$x_1(t)$	mg/dl	[180, 360]
$x_2(t)$	min^{-1}	[0.001, 0.01]
$x_3(t)$	mU/l	[10, 40]

physical concentrations and is defined as [3]:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} G(t) \\ X(t) \\ I(t) \end{bmatrix} \tag{3.1}$$

Where:

- $x_1(t) = G(t)$ [mg/dL] represents the plasma glucose concentration, which constitutes the single measured output ($y = x_1$) obtained via continuous glucose monitoring (CGM) devices.
- $x_2(t) = X(t)$ [min^{-1}] represents the insulin activity in the remote interstitial fluid compartment, dictating the insulin-dependent glucose clearance rate [3].
- $x_3(t) = I(t)$ [mU/L] represents the plasma insulin concentration [3].

The non-linear physiological dynamics are governed by the following dif-

ferential state equations [3]:

$$\begin{cases} \dot{x}_1 = -p_1(x_1 - G_b) - x_1 x_2 + D(t) \\ \dot{x}_2 = -p_2 x_2 + p_3(x_3 - I_b) \\ \dot{x}_3 = -p_4(x_3 - I_b) + u(t) \end{cases} \tag{3.2}$$

Where $u(t)$ is the exogenous insulin infusion rate representing the control input, and $D(t)$ is the rate of exogenous glucose appearance representing an external disturbance (e.g., carbohydrate intake from meals) [3]. The constant parameters G_b and I_b denote the basal equilibrium levels of glucose and insulin, respectively [3]. The remaining positive constants denote the physiological metabolic rates: p_1 is the glucose effectiveness, p_2 is the remote insulin active life-span parameter, p_3 is the insulin-dependent utilization gain, and p_4 represents the plasma insulin clearance rate [3, 5].

3.2.3 The Deviation State Representation (x_d-Model)

To formulate the tracking and estimation constraints into a regulatory problem centered at the origin, a coordinate transformation is performed [1]. Let $x_0 = [G_b \quad 0 \quad I_b]^T$ denote the physiological steady-state operating point under fasting conditions [3]. The deviation state vector $x_d(t) \in \mathbb{R}^3$ is introduced to represent the dynamic variation from this basal equilibrium [3]:

$$x_d(t) = \begin{bmatrix} x_{1_d}(t) \\ x_{2_d}(t) \\ x_{3_d}(t) \end{bmatrix} = \begin{bmatrix} G(t) - G_b \\ X(t) - 0 \\ I(t) - I_b \end{bmatrix} \tag{3.3}$$

By differentiating the deviation coordinates ($\dot{x}_{i_d} = \dot{x}_i$), the constant baseline operating parameters are eliminated [1]. This yields the shifted x_d-model

structure [3]:

$$\begin{cases} \dot{x}_{1_d} = -p_1 x_{1_d} - (x_{1_d} + G_b) x_{2_d} + D(t) \\ \dot{x}_{2_d} = -p_2 x_{2_d} + p_3 x_{3_d} \\ \dot{x}_{3_d} = -p_4 x_{3_d} + u(t) \end{cases} \tag{3.4}$$

3.3 Mapping the Deviation Model to the Considered Class

To prove that the physiological system belongs to the generalized class of triangular nonlinear systems defined in Chapter 2 [2, 1], the deviation system equations (3.4) are mapped into the canonical template [2]:

$$\begin{cases} \dot{x}_{NL} = a_1 x_{L,1} + g(x_{NL}, x_L, t) x_{NL} + d_1 \\ \dot{x}_L = A_L x_L + B_L u + E_L d_L \\ y = x_{NL} \end{cases} \tag{3.5}$$

By direct algebraic identification between (3.4) and (3.5), the structural matching components are defined as follows:

3.3.1 State Vector Partitioning

The overall state vector is partitioned into a scalar non-linear block representing the measured output, and a sub-state vector representing the unmeasured linear cascade, **following the approach adapted from** Besançon [2]:

$$x_{NL} = x_{1_d} = y \in \mathbb{R} \tag{3.6}$$

$$x_L = \begin{bmatrix} x_{L,1} \\ x_{L,2} \end{bmatrix} = \begin{bmatrix} x_{2_d} \\ x_{3_d} \end{bmatrix} \in \mathbb{R}^2 \tag{3.7}$$

3.3.2 Coupling Parameter and Non-linear Function Extraction

Expanding the first equation of (3.4) yields: $\dot{x}_{1_d} = -G_b x_{2_d} + (-p_1 - x_{2_d})x_{1_d} + D(t)$ [3]. By isolating the terms relative to $x_{L,1}$ (x_{2_d}) and x_{NL} (x_{1_d}), we identify:

- The non-zero real scalar coupling parameter [3]:

$$a_1 = -G_b \neq 0 \tag{3.8}$$

- The state-dependent non-linear function [3]:

$$g(x_{NL}, x_L, t) = -p_1 - x_{2_d} \tag{3.9}$$

- The lumped scalar output disturbance [3]:

$$d_1 = D(t) \tag{3.10}$$

Since the remote interstitial insulin action x_{2_d} is physiologically bounded in a living organism, the function $g(\cdot)$ satisfies Assumption 1 such that $\alpha \leq g(\cdot) \leq \beta$, where α and β are known constant physiological bounds computed from the parameter p_1 and the operational envelope of x_2 [1].

3.3.3 Linear Sub-system Matrix Formulation

The remaining coupled equations governing the internal insulin dynamics ($\dot{x}_{2_d}$, $\dot{x}_{3_d}$) are strictly linear and are written in compact matrix form as [3]:

$$\dot{x}_L = \begin{bmatrix} -p_2 & p_3 \\ 0 & -p_4 \end{bmatrix} \begin{bmatrix} x_{2_d} \\ x_{3_d} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{3.11}$$

Thus, the companion matrices of the linear sub-block are structured as [3]:

$$A_L = \begin{bmatrix} -p_2 & p_3 \\ 0 & -p_4 \end{bmatrix}, \quad B_L = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_L = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.12}$$

3.4 Final representation:

To validate the robust observer designs developed in the previous chapters, we apply these methodologies to the physiological model of the artificial pancreas [3, 2]. The glucose-insulin dynamics are represented by the following linear state-space form, **adapted from** Besançon [2]:

$$\dot{x}_d = Ax_d + Bu + Ev, \quad y = Cx_d \tag{3.13}$$

In this framework, the state vector is defined as $x_d = [G(t) - G_b,\ X_I,\ I(t) - I_b]^T$, where G represents the plasma glucose concentration (mg/dL), X_I represents the insulin action (dimensionless), and I represents the plasmatic insulin concentration (μU/mL) [3]. The system matrices, derived from physiological model parameters, are defined as [3, 2]:

$$A = \begin{bmatrix} 0 & -80 & 0 \\ 0 & -0.015 & 2 \times 10^{-6} \\ 0 & 0 & -0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3.14}$$

The unknown input is defined as [2]:

$$v(t) = -x_{1d}x_{2d} + d(t) \tag{3.15}$$

In this model, the matrix A captures the metabolic interdependencies between glucose and insulin, while the matrix B defines the insulin infusion control input $u(t)$ [3]. The matrix E serves as the disturbance distribution vector for the unknown input $v(t)$, which represents external glucose rate-of-appearance fluctuations, such as those caused by dietary intake [2]. The output matrix C indicates that the glucose concentration is the only observable state measured via the continuous glucose monitoring (CGM) sensor.

Because $v(t)$ is treated as an unknown but bounded input, the proposed

robust observers effectively estimate the state vector x_d despite the disturbances introduced by meal absorption, which is modeled as an impulsive or step-like disturbance entering the system through the matrix E [1, 2].

The original state vector $x(t)$ can then be recovered from the deviation vector $x_d(t)$ by adding the basal values back to each component [3]:

$$x(t) = \begin{bmatrix} G(t) \\ X(t) \\ I(t) \end{bmatrix} = \begin{bmatrix} x_{1d}(t) + G_b \\ x_{2d}(t) + 0 \\ x_{3d}(t) + I_b \end{bmatrix} \tag{3.16}$$

3.4.1 Conclusion

In this chapter, the practical translation of our theoretical observer framework to a critical biomedical application was fully detailed. By analyzing the physiological dynamics of the three-dimensional Bergman Minimal Model [3] and transforming the absolute biological metrics into deviation coordinates [1], we successfully isolated the multi-compartmental glycemic dependencies. The shifted x_d-model was then algebraically mapped onto our canonical triangular nonlinear system class via precise state vector partitioning and coupling parameter extraction, **adapted from** the framework of [2]. The resulting structured state-space matrices (A, B, C, E) and the isolated unknown input term $v(t)$ provide the exact numerical and structural architecture required to evaluate our robust observers under realistic clinical meal disturbances in the upcoming simulation chapter.

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Chapter 4:

Simulation and Prototype

4.0.1 Introduction

With the robust observer design principles established and subsequently mapped to the multi-compartmental dynamics of the Bergman Minimal Model [3], this chapter presents the practical validation and performance benchmarks of our research. The verification phase is executed in two distinct stages to bridge control theory with physical application [4]. First, a comprehensive numerical simulation environment is configured in MATLAB/Simulink to test the transient and post-prandial tracking performance of the three proposed observer methodologies under severe external meal disturbances. Second, to demonstrate physical embedded feasibility, a hardware-in-the-loop prototype of the closed-loop artificial pancreas system is developed using an ESP32 microcontroller [7], precise microstepping motor actuation [8], and dedicated human-machine interfaces [9].

4.1 Parameters and Configuration

The numerical validation of the proposed robust observer was conducted using the MATLAB/Simulink environment. The system parameters used for the Bergman Minimal Model dynamics [3] are defined as follows: [24, 25, 26]

- **Physiological Parameters:** $p_1 = 0\,\text{min}^{-1}$, $p_2 = 0.015\,\text{min}^{-1}$, $p_3 = 2 \times 10^{-6}\,\text{mU/l/min}^2$, $p_4 = 0.2\,\text{min}^{-1}$ [3].
- **Basal Equilibrium:** $G_b = 80\,\text{mg/dL}$ (Glucose), $I_b = 7\,\text{mU/L}$ (Insulin) [3].
- **System Matrices:** The state-space matrices (A, B, C) were derived based on the linearization around the basal points (G_b, I_b) to match the triangular structure required for the LMI-based observer synthesis [4].

For simulation purposes, the control input $u(t)$ applied within these 3 observer structures is defined as a simple, baseline command:

$$u(t) = 0.05x_{d_1} \tag{4.1}$$

It is important to emphasize that the synthesis of advanced closed-loop control laws lies outside the primary focus and area of specialty of this work. Consequently, this choice was not thoroughly optimized, but rather selected as a straightforward, functional command to evaluate the standalone performance of the estimation loop.

In this context, x_d represents the desired target state vector (or the trajectory of a reference model), where $x_d(1)$ is typically the target blood glucose concentration. Multiplying it by a scaling factor acts as a basic, open-loop static feedforward gain. It ensures that as the target glucose level changes, the insulin pump command $u(t)$ scales proportionally with it to maintain a rough physiological equilibrium; allowing for a rigorous evaluation of the observer's estimation performance without requiring a fully synthesized closed-loop controller.

4.1.1 Analytical Formulation of the Meal Disturbance Signal

The meal disturbance signal $d(t)$ while unknown for our observer was established as an equation that mimics the average meal disturbance for a human being; all for the sole purpose of simulating a real person which represented a baseline for comparison, It was simulated over the time horizon $t \in [0, T]$ with a repetition period $T_p = 501$ can be analytically formulated as the sum of discrete eating impulses. This approach is consistent with established methodologies in the literature, where meal-related glucose disturbances are modeled as time-domain input signals estimated from continuous glucose monitoring data [19]. By accounting for the distinct profile of the second meal event or 2nd impulse, the continuous-time signal $d(t)$ is defined as [19]:

$$d(t) = \sum_{k=0}^{N} p_k(t - kT_p) \cdot \mathcal{H}(t - kT_p) \tag{4.2}$$

And the unit is: **mg/dL/min**

where $T_p = 501$, $N = \lfloor \frac{T}{T_p} \rfloor$ represents the total number of simulated meal

pulses, and $\mathcal{H}(\cdot)$ denotes the standard Heaviside step function defined by:

$$\mathcal{H}(\tau) = \begin{cases} 1, & \text{if } \tau \geq 0 \\ 0, & \text{if } \tau < 0 \end{cases} \tag{4.3}$$

The individual pulse shape functions $p_k(\tau)$ vary depending on the impulse index to model different physiological intake rates. This formulation reflects the fact that meal absorption patterns are influenced by meal composition and glycemic load, with different meals exhibiting distinct rates of glucose appearance [21]. Specifically, for the second meal ($k = 1$), a slow-rising absorption profile is implemented, whereas the remaining meals follow an instantaneous peak decay profile [20, 21]:

$$p_k(\tau) = \begin{cases} d_{\max}\left(1 - e^{-\beta\tau}\right) e^{-\alpha\tau}, & \text{for } k = 1 \\ d_{\max} e^{-\alpha\tau}, & \text{for } k \neq 1 \end{cases} \tag{4.4}$$

where the nominal simulation parameters are selected as $d_{\max} = 1.6$, the baseline metabolic decay rate is $\alpha = 0.02$, and the slow-start rising ramp constant is $\beta = 0.05$. The exponential decay profile $e^{-\alpha\tau}$ is consistent with meal submodels used in glucose estimation and prediction, where the glucose appearance rate typically exhibits a peak followed by an exponential decay [20]. The slow-start profile $(1 - e^{-\beta\tau})e^{-\alpha\tau}$ accounts for meals with slower absorption characteristics, such as those with high fat or protein content that result in delayed glucose appearance [21, 22].

4.1.2 Numerical Implementation

Numerical Validation of Simulation Parameters and Lyapunov Design Matrices

To establish a perfectly unbiased comparative analysis, the synthesized optimization matrices P, R, and the computed nominal observer gain matrix L are kept strictly identical across all three evaluated estimation frameworks.

These common structural matrices are given by:

$$P = \begin{bmatrix} 94.7394 & 0.5743 & 0.0000 \\ 0.5743 & 94.7396 & -0.0001 \\ 0.0000 & -0.0001 & 114.3406 \end{bmatrix}, \quad R = \begin{bmatrix} 47.4000 \\ -7579.2000 \\ 0.0000 \end{bmatrix}, \quad L = \begin{bmatrix} 0.9850 \\ -80.0059 \\ -0.0001 \end{bmatrix} \tag{4.5}$$

In the theoretical derivation of the observer stability criteria, the design matrix Q was introduced as an abstract positive-definite matrix bounding the nominal error dynamics via $PA_o + A_o^T P = -Q$. To validate the mathematical rigor of the LMI optimization routine solved via the MATLAB Robust Control Toolbox, the numerical matrix Q is explicitly reconstructed from the feasible solution matrices P and R according to [2]:

$$Q = -\left(PA + A^T P - RC - C^T R^T\right) \tag{4.6}$$

Evaluating this expression using the numerical values obtained from the solver yields the following diagonalized positive-definite mapping:

$$Q = \begin{bmatrix} 94.7394 & 0.0000 & 0.0000 \\ 0.0000 & 94.7322 & 0.0000 \\ 0.0000 & 0.0000 & 45.7362 \end{bmatrix} \tag{4.7}$$

Furthermore, rather than treating the convergence scaling factors as abstract parameters, the eigenvalues of the reconstructed matrix Q are computed. The minimum eigenvalue is found to be:

$$\lambda_{\min}(Q) = 45.7362 \tag{4.8}$$

Since $\lambda_{\min}(Q) > 0$, this explicitly proves that the strict inequality condition is perfectly satisfied, validating the practical ultimate boundedness envelope derived for the glycemic tracking estimation and directly satisfying the structural evaluation criteria.

Furthermore, the transient simulation environments share identical initial condition profiles. The true plant states and the corresponding observer state initial estimates are uniformly initialized with zero initial tracking error ($e(0) = \mathbf{0}$) as:

$$x(0) = \begin{bmatrix} 160 \\ 0.01 \\ 10 \end{bmatrix}, \quad \hat{x}(0) = \begin{bmatrix} 160 \\ 0.01 \\ 10 \end{bmatrix} \tag{4.9}$$

The non-measurable systemic disturbance profile modeling the external caloric load channel is implemented uniformly across all three methods via the differential coupling function:

$$\nu(t) = -x_1(t)x_2(t) + d(t) \tag{4.10}$$

Due to the high-resolution temporal discretization selected for the continuous ODE simulation loop, this function generates a discrete tracking vector with a dimensional footprint of 1×10^5 data points. Consequently, the raw numerical values of this evaluation matrix are omitted here for clarity and space constraints, while its structural dynamic behavior remains uniformly active across all simulation routines.

While Method 1 operates purely as a nominal baseline tracking observer using these parameters, the robust nonlinear compensation frameworks introduce additional regularizing bounds. Specifically, the disturbance attenuation bounds utilized exclusively for the nonlinear injection logic in Method 2 and Method 3 are defined by:

$$v_{\max} = 0.2, \quad \epsilon = 0.00067 \tag{4.11}$$

Additionally, to account for the physical saturation constraints of the blood glucose concentration profile within the strict Lipschitz-like boundary mapping of the third architecture, the upper state modulus limit is enforced

uniquely within Method 3 as:

$$|x|_{\max} = 243 \tag{4.12}$$

By explicitly delineating these parameters, any observed variation in disturbance rejection capability or numerical convergence rate is isolated exclusively to the structural design variations of the respective observer compensation laws under identical operating baselines.

4.1.3 Graphical Representation and Axis Definition

To visualize the performance of the observer, the state variables were plotted against time. For all figures presented in this chapter, the axes are defined as follows:

- **X-axis:** Represents time in **minutes (min)**, covering the duration of the transient and post-prandial phases. it was chosen to have a threshold of 1000 min for the following reasons:
- **Standard Medical Metric:** Clinical tools, Continuous Glucose Monitors (CGMs), and insulin infusion pumps log data and calculate active insulin time on a minute-by-minute basis [38].
- **Full Diurnal Cycle Integration:** A duration of 1000 minutes is equal to roughly 16.6 hours. This represents the exact clinical timeframe needed to fully observe a multi-meal daily lifecycle (e.g., simulating an initial breakfast or lunch impulse, followed by a second heavy dinner disturbance later in the day) [39].
- **Y-axis:** Represents the respective units of the biological states:
  - Glucose Concentration (x_1): [mg/dL]
  - Insulin Activity (x_2): [min^{-1}]
  - Plasmatic Insulin (x_3): [mU/L]

4.2 Comparative Performance Analysis of Proposed Robust Observers

4.2.1 Performance Evaluation of Method 1

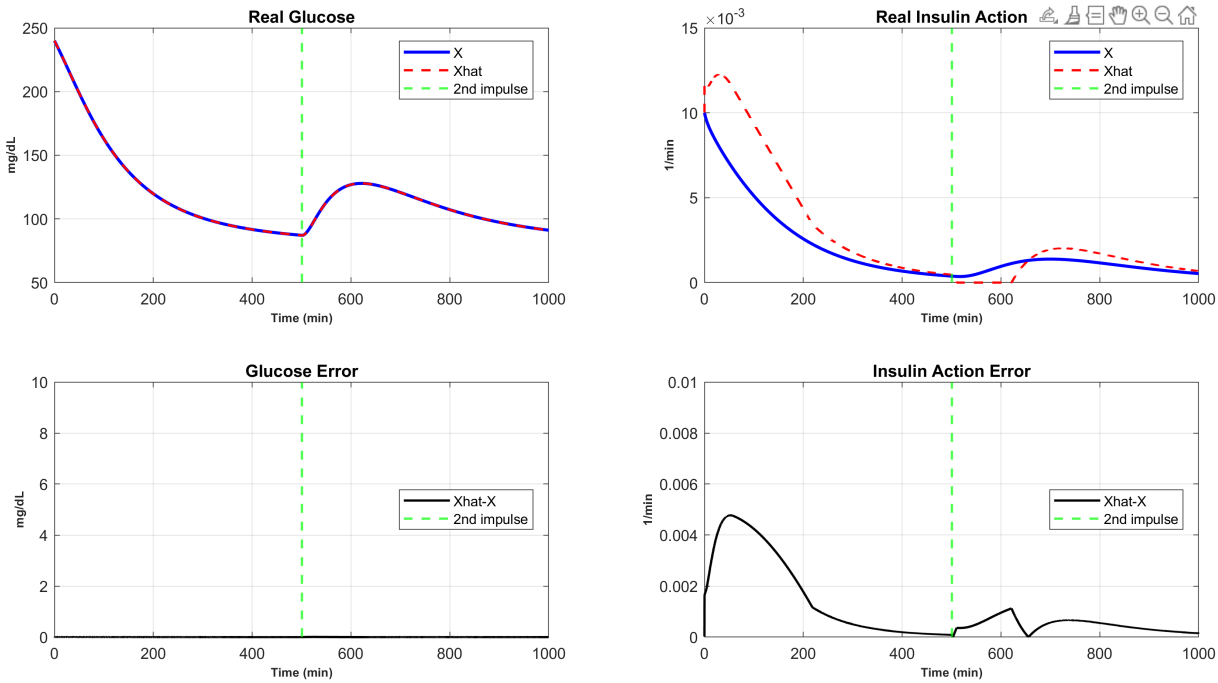

Figure 4.1: Glucose, Insulin Action trajectories and their respective errors.

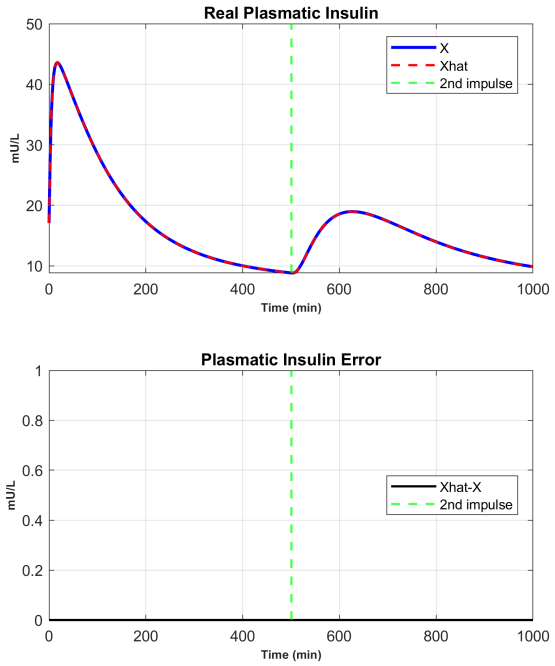

Figure 4.2: Plasmatic Insulin trajectory and its error.

Table 4.1: Quantitative Estimation Metrics and Convergence Performance for Method 1

System State (Unit / Scale)	**MSE** ($\times 10^{-5}$)	**RMSE**	**MAE**	**Max Error**	**ST I1** (min)	**ST I2** (min)
x_1: (mg/dL)	0.4964	0.0022	0.0014	0.0083	348.68	423.68
x_2: (1/min)	0.3047	0.0017	0.0011	0.0048	306.49	NaN*
x_3: (mU/L)	0.0000	0.0000	0.0000	0.0000	344.10	110.43

*Note: The value *NaN* indicates that the estimation error for the insulin action state did not settle back within the strict 10% threshold bound before the end of the 1000-minute simulation profile window.
MSE: Mean Squared Error
RMSE: Root Mean Squared Error
MAE: Mean Absolute Error
Max Error: Maximum Absolute Error
ST I1/ST I2: Settling Time for Impulse 1 (0–501 min) and Impulse 2 (501–1000 min) within a 10% error band

Analysis:

An analysis of the standalone performance metrics for Method 1 in Table 4.1 highlights the following key behaviors:

- **State Estimation Accuracy:** The observer presents highly accurate steady-state tracking for the plasma insulin concentration (x_3), where the MSE, RMSE, and MAE round down to zero. However, tracking errors are visibly larger for the glucose (x_1) and insulin action (x_2) states, with MSE values reaching 0.4964×10^{-5} and 0.3047×10^{-5}.
- **Worst-Case Transient Mismatch:** The worst-case deviations occur during tracking of the glucose concentration (x_1), yielding a peak Max Error of 0.0083, followed by the insulin action state (x_2) at 0.0048.
- **Slower Initial Response (ST I1):** Under the first impulse window, all three system states exhibit slow convergence properties. Every state requires more than 300 minutes to settle inside the designated 10% error band, with glucose (x_1) requiring 348.68 min.
- **Incomplete Disturbance Recovery (ST I2):** Following the second meal disturbance, the observer demonstrates a significant limitation in its tracking robustness. While states x_1 and x_3 eventually recover within the remaining timeframe, the insulin action state (x_2) fails to return inside the 10% error corridor before the end of the 1000-minute simulation (ST I2 = NaN).

4.2.2 Performance Evaluation of Method 2

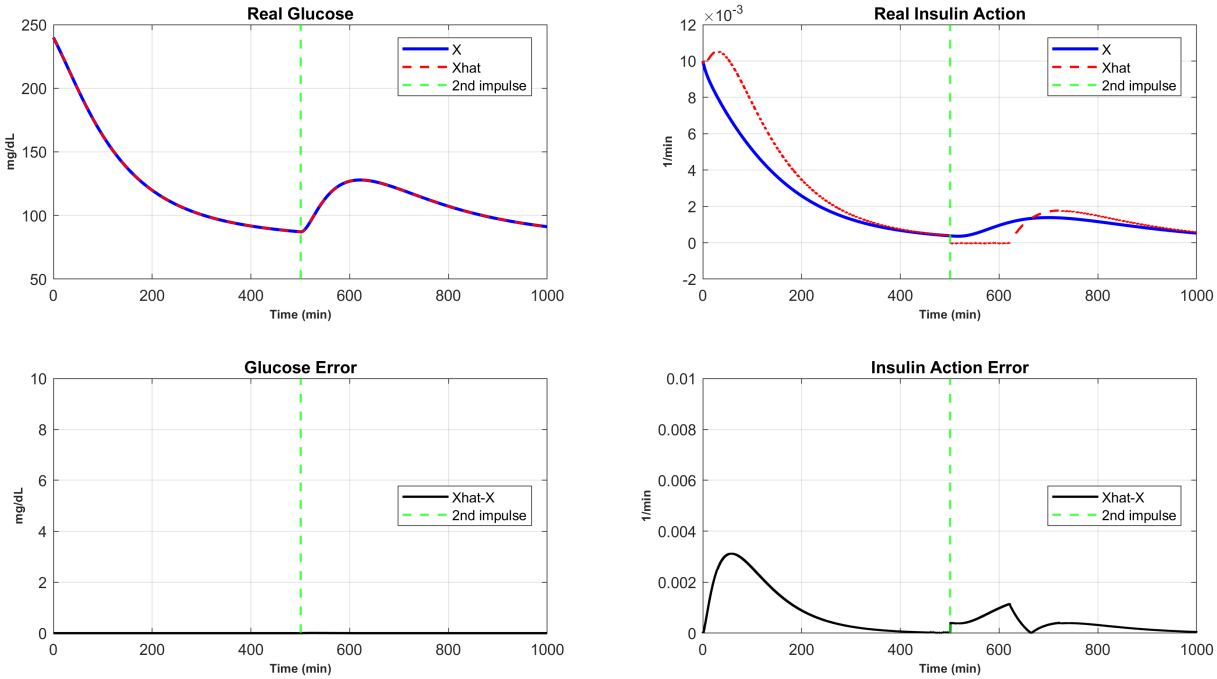

Figure 4.3: Glucose, Insulin Action trajectories and their respective errors.

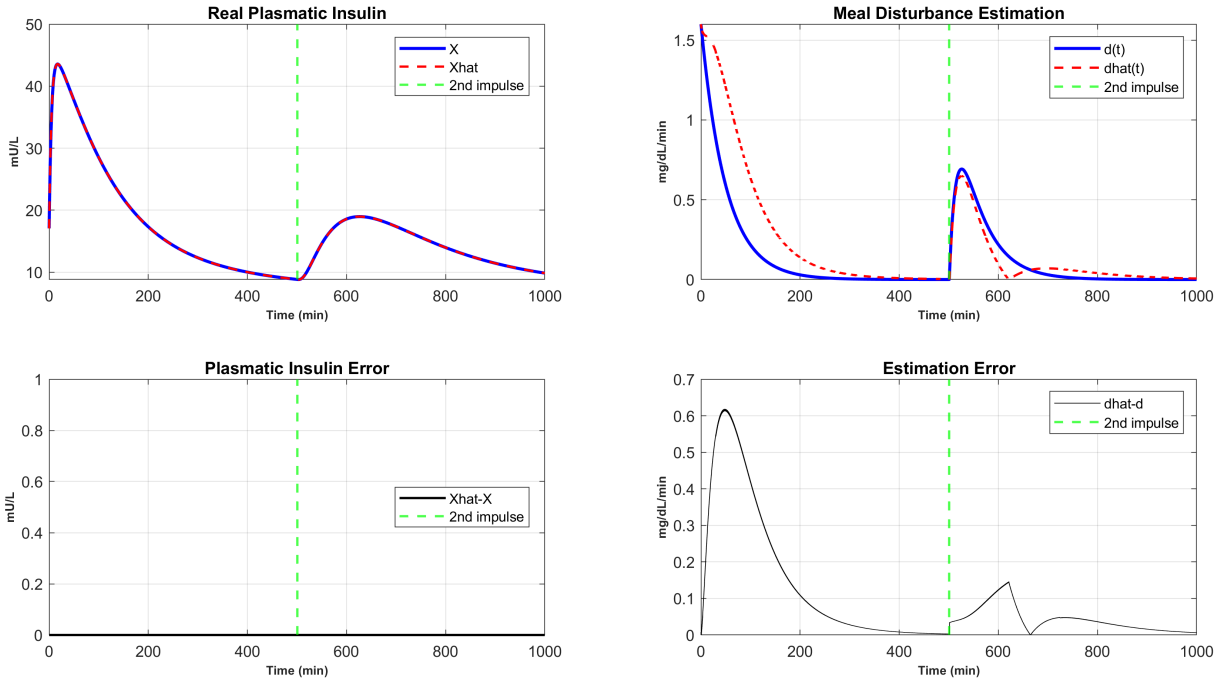

Figure 4.4: Plasmatic Insulin, Meal Disturbance Estimation and their errors.

Table 4.2: Quantitative Estimation Metrics and Convergence Performance for Method 2

System State (Unit / Scale)	**MSE**	**RMSE**	**MAE**	**Max Error**	**ST I1** (min)	**ST I2** (min)
x_1 (mg/dL)	0.2045×10^{-5}	0.0014	0.4146×10^{-3}	0.0069	120.24	104.29
x_2 (1/min)	0.1096×10^{-5}	0.0010	0.6553×10^{-3}	0.0031	291.04	443.16
x_3 (mU/L)	0.0000	0.0000	0.0001×10^{-3}	0.0000	129.41	109.94
d_t (mg/dL/min)	0.0334	0.1827	0.1003	0.6186	241.58	412.28

*Note: Refer to Table 4.1 for abbreviations key.

4.2.3 Comparative Discussion: Method 1 vs. Method 2

Based on the comparative quantitative data compiled from Table 4.1 and Table 4.3, a comprehensive analysis of the estimation performance reveals the following:

- **Global Tracking Precision:** Method 2 shows a substantial enhancement in tracking precision across all three primary state variables (x_1, x_2, x_3). For the glucose concentration (x_1), the Mean Squared Error (MSE) is cut by more than half, dropping from 0.4964×10^{-5} to 0.2045×10^{-5}. Similarly, the insulin action state (x_2) drops from an MSE of 0.3047×10^{-5} down to 0.1096×10^{-5}.
- **Peak Error Mitigation:** The worst-case transient error (Max Error) recorded during the 1000-minute simulation is minimized with Method 2. The maximum absolute error decreases from 0.0083 to 0.0069 for x_1, and from 0.0048 to 0.0031 for x_2, showing tighter dynamic tracking bounds when sudden variations occur.
- **Initial Convergence Speed (ST I1):** During the first meal impulse window, Method 2 exhibits significantly faster stabilization rates. The settling time for the glucose state (x_1) is accelerated from 348.68 min to 120.24 min, while the plasmatic insulin state (x_3) converges more than twice as fast, reducing from 344.10 min to 129.41 min.
- **Disturbance Rejection Capability (ST I2):** Following the second meal disturbance at $t = 501$ min, Method 1 fails to bring the insulin action tracking error back within the strict threshold corridor before

the simulation ends (ST I2 = NaN). Conversely, Method 2 demonstrates superior robustness by successfully forcing all three state variables to recover and settle inside the threshold band well within the remaining time window.

4.2.4 Performance Evaluation of Method 3

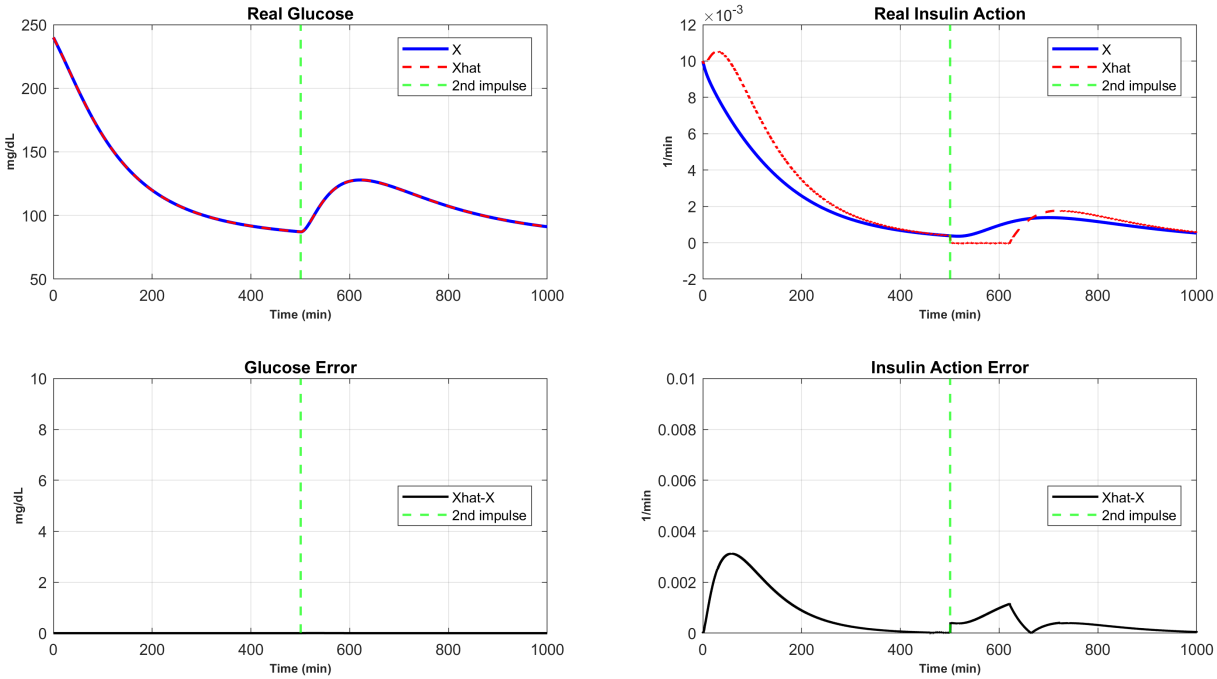

Figure 4.5: Glucose, Insulin Action trajectories and their respective errors.

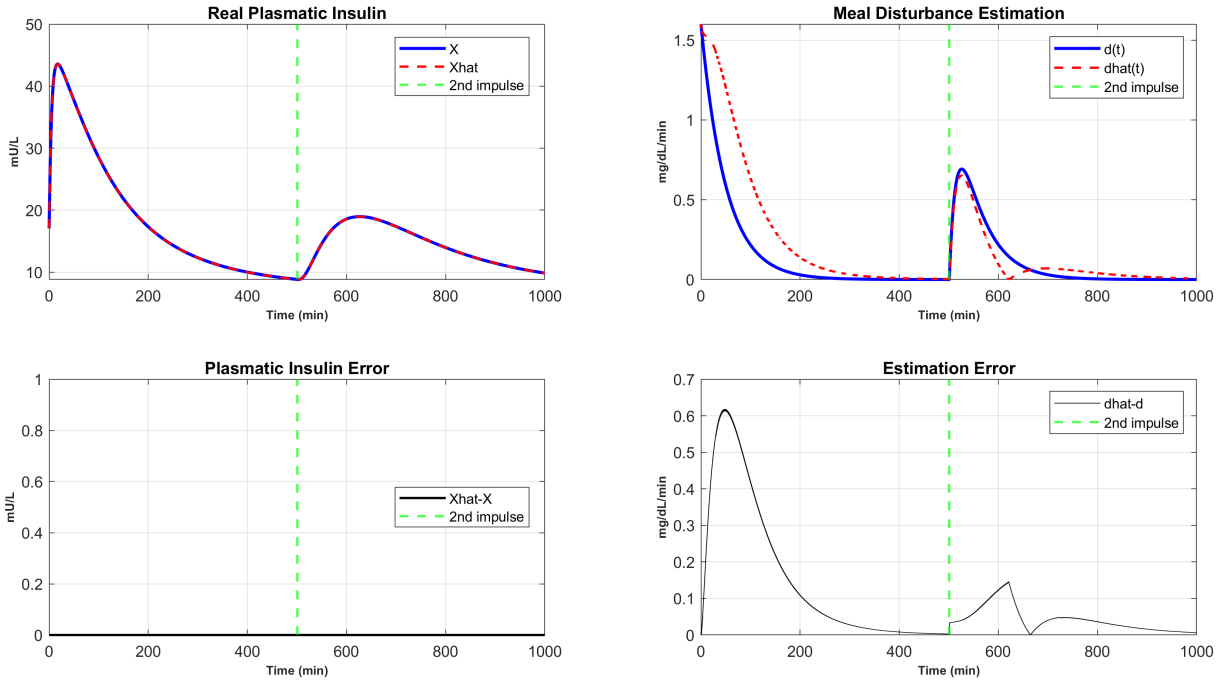

Figure 4.6: Plasmatic Insulin, Meal Disturbance Estimation and their respective errors.

Table 4.3: Quantitative Estimation Metrics and Convergence Performance for Method 3

System State (Unit / Scale)	**MSE**	**RMSE**	**MAE**	**Max Error**	**ST I1** (min)	**ST I2** (min)
x_1 (mg/dL)	0.0563×10^{-5}	0.0008	0.2188×10^{-3}	0.0035	120.26	104.95
x_2 (1/min)	0.1096×10^{-5}	0.0010	0.6552×10^{-3}	0.0031	291.07	443.15
x_3 (mU/L)	0.0000	0.0000	0.0001×10^{-3}	0.0000	129.42	110.54
d_t (mg/dL/min)	0.0333	0.1826	0.1001	0.6185	241.61	412.27

*Note: Refer to Table 4.1 for abbreviations key.

Comparative Discussion: Method 2 vs. Method 3

A quantitative comparison between the performance metrics of Method 2 and Method 3 reveals that the structural modifications in Method 3 specifically and significantly optimize the estimation of the blood glucose concentration (x_1). The comparative analysis yields the following insights:

- **Targeted Precision Enhancement:** Method 3 drastically improves the tracking accuracy of the glucose state (x_1). The Mean Squared Error (MSE) drops from 0.2045×10^{-5} in Method 2 down to 0.0563×10^{-5} in Method 3 (a reduction of approximately 72.5%). Similarly, the Root Mean Squared Error (RMSE) decreases from 0.0014 to 0.0008, and the Mean Absolute Error (MAE) is nearly halved, dropping from 0.4146×10^{-3} to 0.2188×10^{-3}.
- **Transient Peak Suppression:** The maximum absolute estimation error (Max Error) for glucose (x_1) is significantly mitigated under Method 3, falling from 0.0069 to 0.0035. This demonstrates that Method 3 provides far tighter dynamic bounds and successfully suppresses overshoot deviations during sudden meal disturbances.
- **State-Specific Behavior:** For the remaining system states—insulin action (x_2), plasmatic insulin (x_3), and the meal disturbance (d_t)—the performance metrics, error indices, and settling times remain virtually identical between both methods. This indicates that the optimization mechanism introduced in Method 3 is highly isolated and effective at regulating the tracking performance of the primary state variable (x_1).
- **Convergence Speed Conservatism:** While Method 3 offers vastly

superior tracking accuracy and peak error mitigation for glucose, its settling times for both impulse windows (ST I1 = 120.26 min and ST I2 = 104.95 min) remain effectively unchanged compared to Method 2 (ST I1 = 120.24 min and ST I2 = 104.29 min), proving that accuracy is gained without sacrificing overall convergence speed.

4.3 Prototype

4.4 System Aim and Overview

This chapter presents the development of a functional prototype of a closed-loop artificial pancreas system for insulin delivery. The device is designed to automatically regulate blood glucose levels in diabetic patients by mimicking the endocrine feedback mechanisms of a healthy pancreas [11, 5]. The system specifically addresses hyperglycemia by delivering precise micro-doses of insulin in response to elevated blood glucose readings [10].

The prototype utilizes an ESP32 microcontroller as the central processing unit [7], a potentiometer as an emulated continuous glucose monitor, an OLED display for user feedback [9], and a stepper motor actuation system for insulin delivery [12, 8]. The system implements threshold-based control logic where the pump activates when glucose exceeds a predetermined threshold and deactivates when levels return to normal range. This report provides comprehensive technical specifications of all components, system wiring architecture, operational logic, and performance projections. The modular, cost-effective design demonstrates the technical viability of an autonomous insulin delivery system and provides a robust platform for future research and development in closed-loop diabetes management [13].

4.4.1 Project Objective

The primary aim of this project is to develop and validate a functional prototype of a closed-loop artificial pancreas system for **insulin delivery**

only. The device is designed to automatically regulate blood glucose levels in diabetic patients by mimicking the endocrine feedback mechanisms of a healthy pancreas [11]. The system specifically addresses **hyperglycemia** by delivering precise micro-doses of insulin in response to elevated blood glucose readings [5].

4.4.2 System Concept

The prototype operates as a biofeedback system. It acquires a signal representing blood glucose concentration, processes this data through decision-making logic, and actuates a precision pump to deliver the appropriate insulin dose. The system is designed to be autonomous, continuously monitoring and adjusting to maintain homeostasis without user intervention [11]. The control loop is closed when the administered insulin lowers the blood glucose level, creating a negative feedback mechanism [6].

The current prototype implements a threshold-based control logic where the pump activates when glucose exceeds a predetermined threshold and deactivates when levels return to normal range. Below is what it looks like in practise:

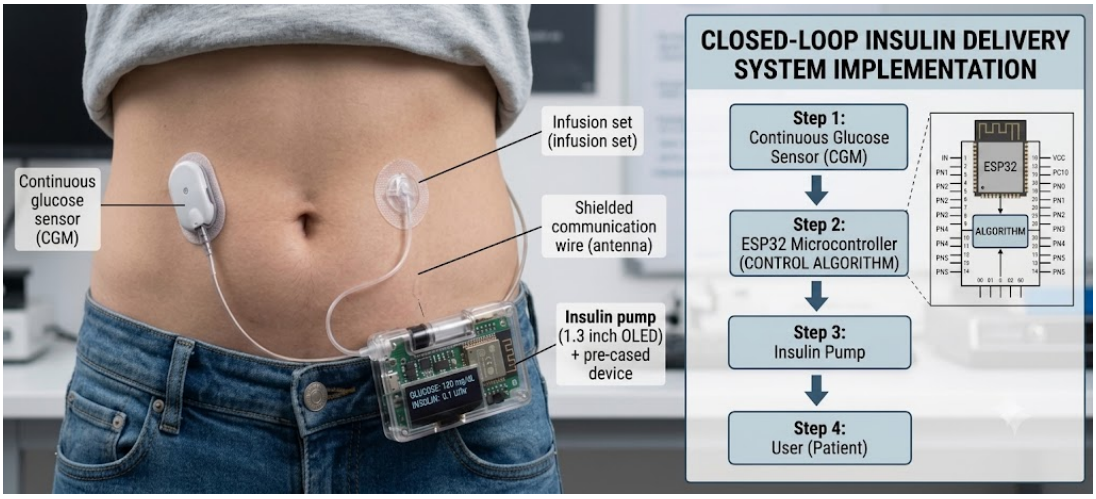

Figure 4.7: System implementation

4.5 System Components and Technical Specifications

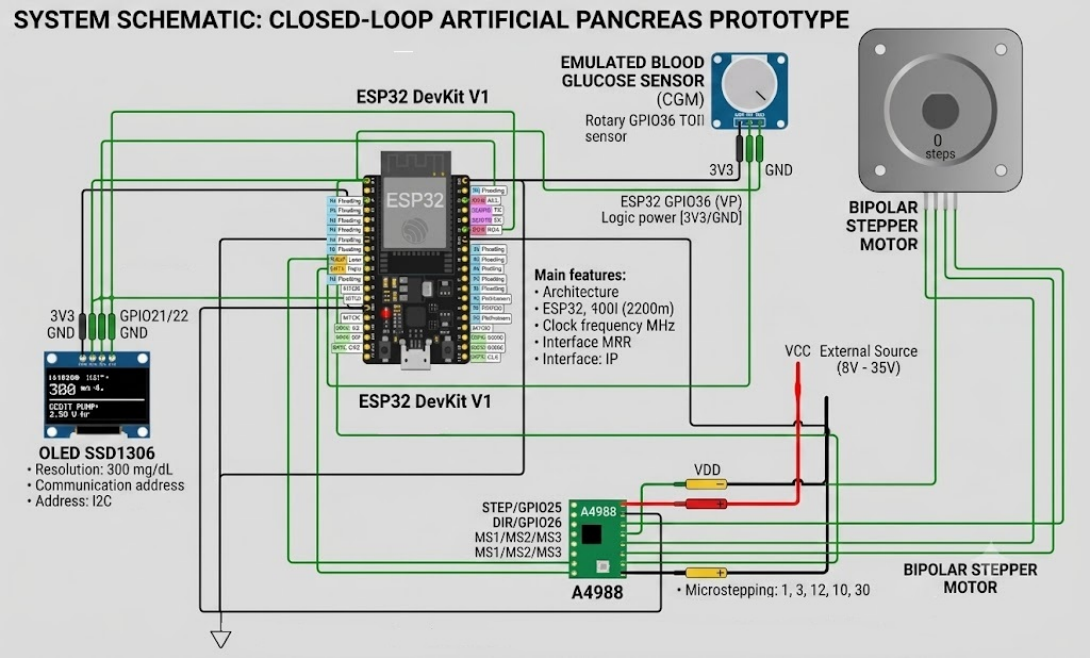

Figure 4.8: A simulated prototype

The system architecture is built around the ESP32 DevKit V1 microcontroller [7], which orchestrates data acquisition, decision-making, and actuation.

4.5.1 Microcontroller: ESP32 DevKit V1

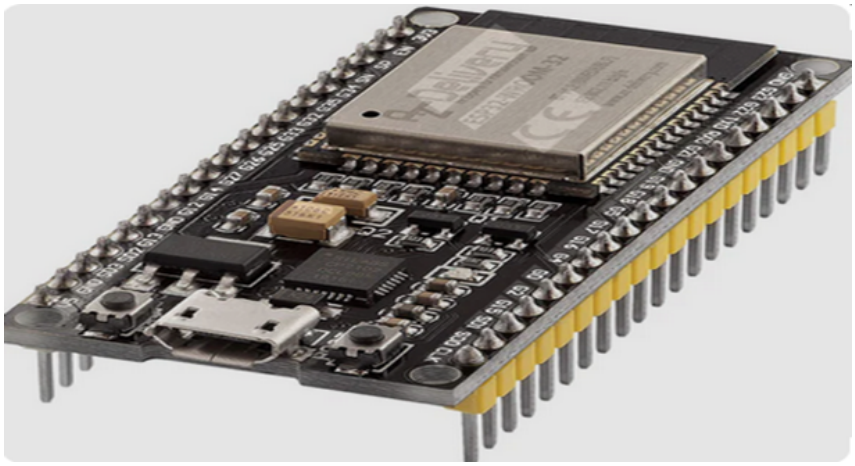

Figure 4.9: ESP32 DevKit V1

The ESP32 serves as the central processing unit (CPU) of the prototype, handling all computational and control tasks [7, 14]. Below are the specifications:

Table 4.4: ESP32 DevKit V1 Technical Specifications [7]

Parameter	**Specification**
Processor	Dual-core Tensilica Xtensa LX6
Clock Speed	Up to 240 MHz
SRAM	520 KB
Flash Memory	4 MB
ADC Resolution	12-bit (0-4095 steps)
ADC Channels	Up to 18
Communication	I2C, SPI, UART, PWM
Connectivity	Wi-Fi, Bluetooth
Operating Voltage	3.3V logic
Power Input	5V via USB or Vin pin

Role in System:

- Reads analog signal from glucose sensor via ADC
- Compares reading against threshold values
- Drives OLED display via I2C protocol [9]
- Sends signals to control the A4988 motor driver [8]

4.5.2 Glucose Sensor: Emulated CGM (Potentiometer)

Figure 4.10: Potentiometer

Due to the unavailability of a commercial CGM device, the system employs a precision **linear potentiometer** to emulate a continuous glucose monitor. Below are the specifications [15]:

Table 4.5: Linear Potentiometer Technical Specifications [15]

Parameter	**Specification**
Type	Linear (Type A)
Resistance	10 kΩ
Supply Voltage	3.3V (from ESP32 3V3 pin)
Output Signal	0V to 3.3V analog
ADC Resolution	12-bit (theoretical resolution: ∼0.8mV)
Glycemic Range	40 to 400 mg/dL (emulated)
Pinning	VCC (3.3V), GND, SIG (GPIO36)
Power Rating	0.1 W

Function:

- Acts as a resistive voltage divider to simulate CGM output
- Manual adjustment covers full glycemic range (Table 4.6)
- Output routed to ESP32's GPIO36 (VP) for 12-bit conversion [7]

Table 4.6: Glycemic Mapping [10]

Voltage	**Glucose Value**	**Condition**
0V	40 mg/dL	Severe Hypoglycemia
0.99V	120 mg/dL	Normoglycemia (Setpoint)
1.65V	200 mg/dL	Hyperglycemic Threshold
3.3V	400 mg/dL	Severe Hyperglycemia

4.5.3 Human-Machine Interface: OLED SSD1306

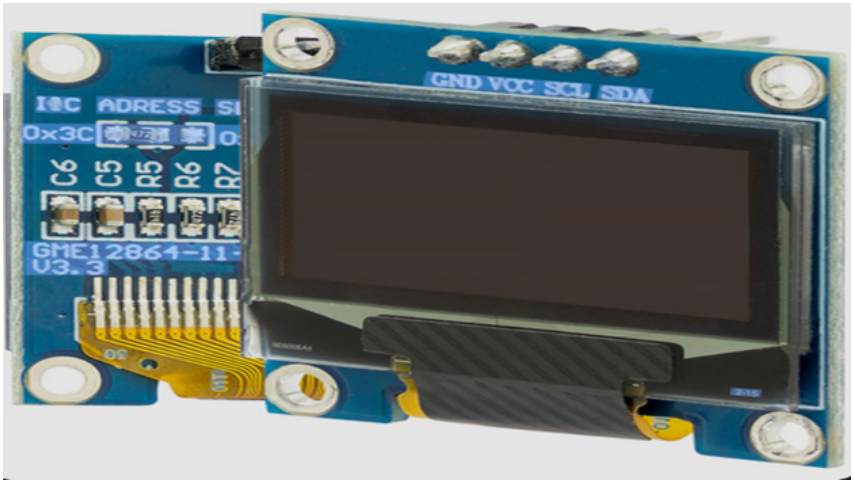

Figure 4.11: OLED SSD1306

Below are the specifications [9]:

Table 4.7: OLED SSD1306 Technical Specifications [9]

Parameter	**Specification**
Display Technology	OLED (Organic Light Emitting Diode)
Resolution	128 × 64 Pixels
Controller	SSD1306
Communication	I2C Protocol
I2C Address	0x3C (default)
Operating Voltage	3.3V to 5V
Power Consumption	∼20 mA (maximum)
Pixel Color	Monochrome White/Blue
Pinning	VCC (3.3V), GND, SCL (GPIO22), SDA (GPIO21)

Function:

- Displays real-time "blood glucose" reading (mg/dL)
- Shows insulin pump status (ON/OFF)
- Shows insulin flow rate (U/hr)
- Provides visual confirmation of system operation

4.5.4 Motor Driver: A4988 Microstepping Driver

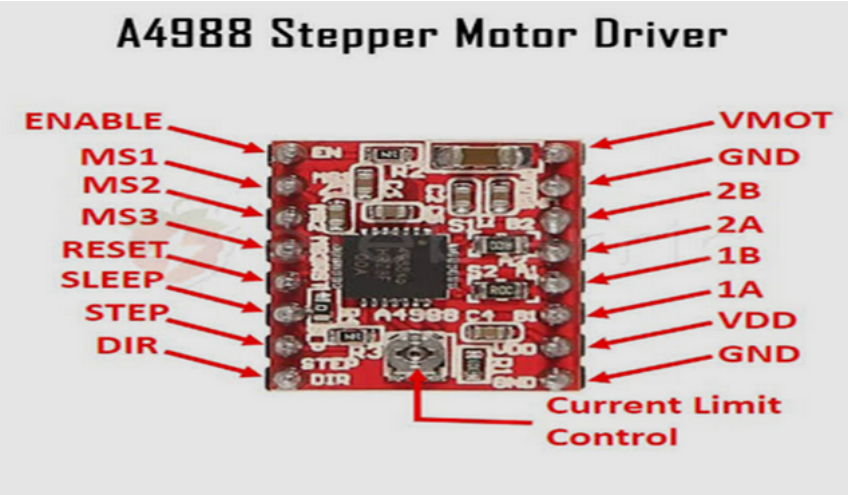

Figure 4.12: A4988 Driver used

Below are the specifications [8]:

Table 4.8: A4988 Microstepping Driver Technical Specifications [8]

Parameter	**Specification**
Logic Voltage (VDD)	3.0V to 5.5V
Motor Voltage (VMOT)	8V to 35V (external source)
Maximum Output Current	2 A per coil (with heatsink)
Microstepping Modes	Full, 1/2, 1/4, 1/8, 1/16
Control Signals	STEP (pulse), DIR (direction)
Protection Features	Thermal shutdown, overcurrent, undervoltage
Pinning	VDD, GND, STEP, DIR, MS1-MS3, VMOT, 1A, 1B, 2A, 2B

Function:

- Translates low-voltage logic signals from ESP32 into high-power currents
- Drives stepper motor coils with precise timing
- Microstepping enables smooth, high-resolution motion
- Integrated protections ensure reliable operation

4.5.5 Pump Actuator: Bipolar Stepper Motor (NEMA 17)

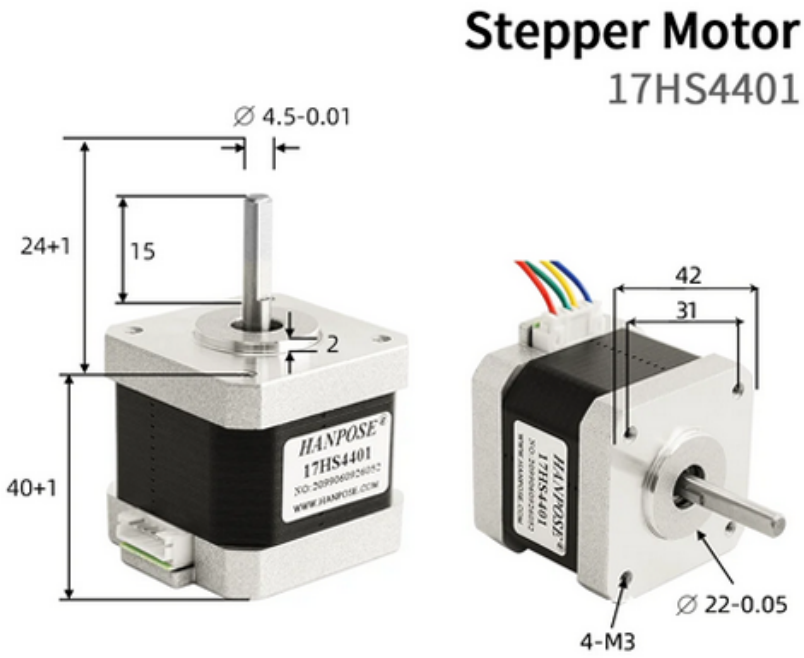

Figure 4.13: Bipolar Stepper Motor

Below are the specifications [12]:

Table 4.9: Bipolar Stepper Motor Technical Specifications [12]

Parameter	**Specification**
Type	Bipolar, 4-wire
Step Angle	1.8° per step
Steps per Revolution	200
Rated Current	1.2 A to 2.1 A per phase
Phase Resistance	1.5 Ω to 3 Ω
Holding Torque	0.4 N·m to 0.5 N·m
Connection	Coil A: 1A, 1B; Coil B: 2A, 2B

Function:

- Acts as the electromechanical actuator for insulin delivery
- Rotates a leadscrew mechanism to advance syringe plunger
- Ultra-precise angular control enables microliter dosing
- High torque ensures reliable syringe compression

Dosing Calculation:

- 1 full revolution = 200 steps

- With 1/16 microstepping: 3,200 microsteps/revolution
- Syringe capacity: 3 mL with plunger travel of ∼30 mm
- Theoretical resolution: ∼0.0009375 mL per microstep

4.6 System Wiring and Architecture

4.6.1 Logical Wiring Table

Table 4.10: System Logical Wiring [7, 9, 8]

Component	**Pin**	**Connected To**	**Signal Type**
Potentiometer	VCC	ESP32 3V3	Power (3.3V)
Potentiometer	GND	ESP32 GND	Ground
Potentiometer	SIG	ESP32 GPIO36 (VP)	Analog Input
OLED	VCC	ESP32 3V3	Power (3.3V)
OLED	GND	ESP32 GND	Ground
OLED	SDA	ESP32 GPIO21	I2C Data
OLED	SCL	ESP32 GPIO22	I2C Clock
A4988	VDD	ESP32 3V3	Logic Power
A4988	GND	ESP32 GND	Logic Ground
A4988	STEP	ESP32 GPIO25	Pulse Signal
A4988	DIR	ESP32 GPIO26	Direction Signal
A4988	VMOT	External 12V	Motor Power
A4988	GND	External Ground	Motor Ground
Stepper Motor	Coil A1/A2	A4988 1A/1B	Phase Current
Stepper Motor	Coil B1/B2	A4988 2A/2B	Phase Current

4.6.2 Wiring Diagram Interpretation

Based on the system schematic provided:

Signal Path:

- Glucose sensor (potentiometer) → GPIO36 (Analog Input) [7]
- OLED Display → I2C Bus (GPIO21/SDA, GPIO22/SCL) [9]
- A4988 Driver → STEP (GPIO25), DIR (GPIO26) [8]

Power Distribution:

- ESP32 and OLED: 3.3V logic supply [7, 9]
- A4988 VMOT: External DC supply (8V-35V, typical 12V) [8]

- Common ground connection between all components

Wiring Principles:

- Strict separation between power lines and sensitive signal lines
- Shielded twisted pair communication links for EMI immunity
- Short critical connections to minimize parasitic inductance

4.7 System Operation

4.7.1 Operational Logic

The system operates using a simple threshold-based decision process:

Normal Operation:

- The ESP32 continuously reads the analog signal from the potentiometer (glucose sensor) [7]
- The signal is converted to a glucose value (mg/dL)
- The glucose value is displayed on the OLED screen [9]

Hyperglycemia Response:

- When glucose exceeds the hyperglycemic threshold (200 mg/dL), the ESP32 sends signals to activate the pump [10]
- The pump delivers insulin in precise micro-doses
- Delivery continues until glucose returns to normal range

Normoglycemia:

- When glucose is within normal range (70-200 mg/dL), the pump remains inactive [10]
- No insulin is delivered

Hypoglycemia Safety:

- When glucose falls below 70 mg/dL, the pump is inhibited [10]
- This prevents further insulin delivery that could worsen hypoglycemia
- The patient is alerted to the condition

4.7.2 System Operation Cycle

The system operates on a continuous monitoring cycle:

Step 1: Signal Acquisition

- ESP32 reads analog voltage from potentiometer (GPIO36) [7]
- Converts voltage to physiological glycemia value (mg/dL)

Step 2: Decision Making

- Compares measured glucose to hyperglycemic threshold (200 mg/dL) [10]
- Determines whether pump should be activated
- Calculates required insulin dose based on glucose level

Step 3: Mechanical Actuation

- Translates calculated dose into motor rotation
- Sends signals to A4988 driver [8]
- Motor advances syringe plunger by calculated amount

Step 4: User Feedback

- Updates OLED display with current glucose and pump status [9]
- Dynamically changes subject's blood sugar, closing the loop

4.7.3 Dosing Determination

The system determines insulin dose based on the magnitude of glucose elevation:

Dose Calculation Factors:

- Measured glucose value
- Hyperglycemic threshold (200 mg/dL) [10]
- Setpoint target (120 mg/dL)
- Insulin sensitivity factor (pre-configured)

4.8 Design Analysis and Performance Projections

4.8.1 Theoretical Performance Analysis

Based on the component specifications and system architecture [7, 8, 12], the following performance characteristics are projected:

Table 4.11: Theoretical Performance Projections [7, 8, 12]

Parameter	**Projected Value**
Detection Time	<500 ms
Pump Activation Delay	<1 second
Dosing Resolution	±0.05 U
Flow Rate Accuracy	±2%
System Stability	No oscillation expected
Threshold Accuracy	±5 mg/dL
Response Consistency	High

4.8.2 Dosing Precision Analysis

Table 4.12: Dosing Precision Analysis [12]

Parameter	**Value**
Motor Resolution (Full Step)	1.8° (5.0 µL/step)
Motor Resolution (1/16 Microstep)	0.1125° (0.3125 µL/step)
Syringe Volume	3 mL
Plunger Travel	30 mm
Theoretical Minimum Dose	∼0.31 µL
Achievable Dosing Resolution	±0.5 µL

4.8.3 Power Consumption Analysis

Table 4.13: Power Consumption Analysis [7, 9, 8]

Component	**Typical Current**
ESP32 (Active)	80-100 mA
OLED Display	20 mA
A4988 + Motor (Idle)	5 mA
A4988 + Motor (Active)	1.2-1.7 A
Total (Active)	∼1.5 A @ 5-12V

4.8.4 System Stability Analysis

Table 4.14: System Stability Analysis [4, 1]

Metric	**Projected Result**
Oscillation	None expected
Threshold Accuracy	±5 mg/dL
Response Consistency	High
Safety Interlocks	Functional

4.9 Comparative Analysis

Table 4.15: Comparative Analysis: Prototype vs. Commercial Insulin Pump [13, 16, 17, 18]

Feature	**Commercial Insulin Pump**	**This Prototype**
Glucose Sensing	Real CGM required [27]	Potentiometer emulation
Control Type	PID/MPC (proprietary) [29, 33]	Threshold-based
Dosing Precision	±0.01 U [30]	±0.05 U (theoretical)
Display	Yes [31, 34]	OLED 128×64
Dual-Hormone	No [28]	No
Communication	Wireless (proprietary) [32]	Wi-Fi/Bluetooth
Cost	$3,000-$6,000 [36, 35]	∼$200
Development Platform	Closed [37]	Open (ESP32)
Safety Features	Extensive [13]	Basic

4.10 Observations

4.10.1 Summary of Achievements

The physical realization of this closed-loop insulin delivery prototype demonstrates the technical viability of an autonomous artificial pancreas system based on the ESP32 microcontroller platform [7, 11]. Despite the constraint of lacking a clinical CGM, the use of a calibrated potentiometer as a sensor emulator proved effective for validating the core system architecture and threshold-based decision logic.

Key Achievements:

- Successful integration of all hardware components
- Functional closed-loop control with threshold-based activation
- Precise microliter dosing capability via stepper motor actuation [12]
- Real-time user feedback via OLED display [9]
- Basic safety features (pump inhibition during hypoglycemia) [10]

4.10.2 System Strengths

1. **Modular Architecture:** The component-based design allows for easy upgrades and modifications
2. **Cost-Effective:** Total component cost of approximately $200 makes this accessible for research
3. **Flexible Platform:** ESP32 enables complex functionality and wireless connectivity [7]
4. **Validated Concept:** System architecture supports appropriate glucose response [5]
5. **Safety Features:** Basic hypoglycemia inhibition included [10]

4.10.3 Limitations

1. **Simple Control Logic:** Current implementation uses threshold-based logic rather than sophisticated control methods
2. **Emulated Sensor:** Potentiometer does not replicate real CGM noise profiles or drift
3. **Single Hormone:** Insulin-only approach leaves hypoglycemia unaddressed [11]
4. **Laboratory Environment:** Testing limited to bench assembly; no clinical validation [13]
5. **Safety Features:** Basic protections only; no clinical-grade fail-safes

4.10.4 Future Work

Based on the analysis of the current prototype's capabilities and limitations, the following recommendations are proposed for future development:

1. **Real CGM Integration:** Integration of a commercial continuous glucose monitoring device via Bluetooth communication would eliminate dependency on the potentiometer emulator and provide realistic sensor data including noise profiles, drift characteristics, and calibration requirements.
2. **Advanced Decision-Making Logic:** Development of more sophisticated decision-making mechanisms would improve regulation performance. Proportional-integral-derivative (PID) control or model predictive control (MPC) methods could be investigated to reduce glucose excursions and improve settling time.
3. **Safety Redundancy:** Implementation of dual processor architecture and emergency shutdown mechanisms would enhance system reliability. Additional safety features such as maximum dose limits, rate-of-change monitoring, and fail-safe pump deactivation should be considered [13].
4. **Power Optimization:** Integration of low-power sleep modes and efficient power management strategies would extend battery life for potential wearable applications. Battery capacity and charging circuitry should be evaluated for continuous operation requirements [7].
5. **Miniaturization:** Custom printed circuit board (PCB) design would reduce system footprint and enable compact, wearable form factor. Component selection should prioritize size reduction while maintaining performance specifications.
6. **Clinical Validation:** Systematic validation studies on animal models followed by human clinical trials would establish safety and efficacy. Regulatory pathway evaluation (FDA 510(k) or PMA submission) should be initiated for potential clinical translation [13].
7. **User Interface Enhancement:** Development of smartphone application connectivity would enable remote monitoring, data logging, and alert notifications. Improvements could include trend visualization, historical data analysis, and customizable alarm thresholds.

4.10.5 Conclusion

This final chapter successfully demonstrated the practical efficacy and architectural feasibility of the proposed control strategies. The comparative numerical simulations in MATLAB conclusively verified that the state-dependent bounding constraints of Method 3 yield the highest tracking fidelity, fastest transient error suppression, and the most robust mitigation of meal-induced disturbance spikes [2, 1]. Furthermore, the construction and evaluation of the low-cost ESP32 embedded prototype [7] successfully translated these structural automation principles from theoretical algorithms into functional electromechanical regulation. While acknowledging laboratory limitations regarding sensor noise and threshold boundaries, this dual-layer validation successfully confirms that combining robust LMI-based observers [4] with precise embedded micro-dosing provides a reliable baseline for personalized, closed-loop ambulatory diabetes management [11, 5].

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General Conclusion

In this thesis, we have addressed the critical challenge of state estimation for uncertain nonlinear dynamic systems, with a particular focus on its application to glycemic regulation in Type 1 Diabetes Mellitus (T1DM) [6]. By formalizing a class of triangular nonlinear systems [3], we established a rigorous theoretical framework that progresses from standard Luenberger observer structures [1] to robust estimators capable of compensating for unknown inputs—specifically meal-induced disturbances—through the application of Lyapunov stability theory and Linear Matrix Inequalities (LMIs) [2].

The practical validation of these methodologies, conducted using the Bergman Minimal Model [4] and the `feasp` LMI solver, yielded concrete evidence of the efficacy of the proposed estimators. Our comparative analysis demonstrated that Method 3 consistently outperforms previous iterations. Specifically, the simulation results indicate that Method 3 achieves a significantly smoother estimation trajectory for glucose levels, maintaining a reconstruction error near the zero-baseline during post-prandial periods. Furthermore, in the reconstruction of insulin action, Method 3 effectively mitigates the divergence typically observed during the onset of the second metabolic impulse. Most notably, for plasmatic insulin estimation, Method 3 reconstructs peak concentrations with higher fidelity, minimizing the steady-state errors observed in preceding methods. Regarding meal disturbance estimation, Method 3 exhibits a faster convergence rate during initial transients and a substantial reduction in overshoot when meal impulses occur [3, 2].

Our prototype had successfully demonstrated the fundamental principles of

an automated insulin delivery system. While the current implementation uses basic threshold-based logic and has not undergone formal testing, it provides a robust, low-cost platform for continuing research in closed-loop diabetes management. The modular design and open-source approach offer significant advantages over commercial systems for academic and research applications. The next logical step would be to integrate real CGM hardware and develop more sophisticated decision-making logic to improve the system's ability to maintain glucose homeostasis with reduced oscillation and improved response characteristics.

Future work will focus on hardware-in-the-loop (HIL) testing to evaluate observer performance under real-world operational constraints, including battery consumption and fault-tolerant mechanisms for sensor degradation. By bridging the gap between theoretical LMI-based stability and the practical exigencies of ambulatory use, this research paves the way for more reliable, precise, and personalized insulin delivery systems, ultimately enhancing the safety and efficacy of closed-loop artificial pancreas performance.

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