Available online at www.sciencedirect.com
ScienceDirect
Solid State Ionics xx (2008) xxx–xxx
SOLID STATE IONICS
www.elsevier.com/locate/ssi

United Gauss–Pearson-IV distribution model of ions implanted into silicon ☆

Haipeng Zhang *, Mingyu Gao, Liyan Xu, Mi Lin, Xiaoyan Niu, Weifeng Lv

School of Electronics and Information, Hangzhou Dianzi University, Hangzhou, 310018, China

Received 29 August 2007; received in revised form 21 January 2008; accepted 26 January 2008

Abstract

In this paper a united Gauss–Pearson-IV (UGP) distribution model is presented by definition of a weight factor R_g, which represents the percent rate between the number of channeling ions and the number of all ions implanted for the first time based on determination of some basic relation ships and correction of some basic conclusions of Pearson-IV distribution model. The UGP models fit the experimental results of high energies B ions implantation into crystal Si with and without oxide mask and low energy BF_2 implantation into crystal and amorphous Si very well.

Keywords: Ion implantation; Silicon; UGP model; Channeling effect; Random scattering effect

1. Introduction

Ion implantation is a low temperature technique for introducing impurities into semiconductors and offers more flexibilities than diffusion. The projected ranges of impurities implanted into silicon could be controlled precisely. Whether the ions are implanted into silicon very shallow or deep, their profiles can be part-precisely depicted by semi-Gauss, Pearson-IV distribution or dual Pearson-IV distribution [1]. It will continue to be the primary means of introducing the impurity atoms into the semiconductor device structures as the semiconductor device decreases in size [2]. However, little distribution functions have been developed for accurately picturing the dumb bell type distribution when impurities are implanted neither very shallow nor very deep or are implanted twice with distinctively different depths into silicon.

2. United Gauss–Pearson-IV distribution

In this paper we present a united Gauss–Pearson-IV (UGP) distribution model by definition of a weight factor R_g, which represents the percent rate between the number of channeling ions and the number of all ions implanted for the first time.

2.1. Gauss distribution

When the radius of implanted ions is big and the implantation energy is low enough or the implantation is carried out with an amorphous mask and a proper tilt angle, R_g approaches 0, the number of channeling ions can be neglected so that the impurity profile can be pictured precisely enough with Gauss distribution:

$$f(z) = \frac{1}{\sqrt{2\pi}\Delta R_p} \exp\left(-\frac{1}{2}\cdot\left(\frac{Z-R_p}{\Delta R_p}\right)^2\right), \quad (1)$$

where, R_P represents the projected range, ΔR_P is the standard deviation of projected range and $R_p > \sqrt{2}\Delta R_p$. A group of normalized p.d.f.s of Gauss distribution are illustrated in Fig. 1 with the characteristic parameters of $R_p = 0.5$, $\Delta R_{p1} = 0.05$, $\Delta R_{p2} = 0.1$, and $\Delta R_{p3} = 0.2$.

The Gauss distribution is a kind of normal distributions and is very suitable for description of random scattering component in ions implantation fields.

☆ Project financially supported by the National Natural Science Foundation of China (grant number: 60306003), the Natural Science Foundation of Zhejiang Province (grant number: y104599) and the open fund of circuit and system as one of the most important subjects in Zhejiang province (grant number: ZZ050103-7).

* Corresponding author. Tel.: +86 13093725207; fax: +86 571 86919161.
E-mail address: islotus@163.com (H. Zhang).

0167-2738/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.ssi.2008.01.084

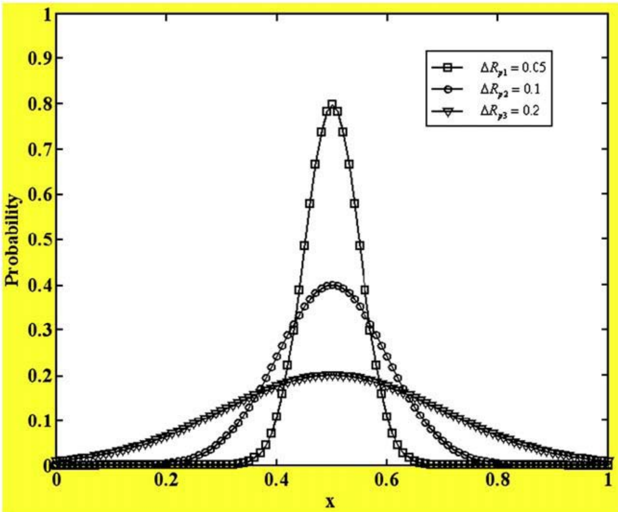

Fig. 1. A group of normalized p.d.f.s of Gauss distribution.

2.2. Pearson-IV distribution

As the radius of implanted ions is small and the implantation energy is high enough on the conditions that the implantation is carried out with a small tilt angle and without an amorphous mask, R_g approaches 1, which means that channeling effect is predominant so that the impurity profile could be pictured with Pearson-IV distribution:

$$\frac{dh(z_R)}{dz_R} = \frac{(z_R - a_4)h(z_R)}{b_2 z_R^2 + b_1 z_R + b_0}, \tag{2}$$

where,

$$z_R = z - R_{p-4}, \tag{3}$$

$$a_4 = b_1 = -\gamma \Delta R_{p-4}(\beta + 3)/B_3, \tag{4}$$

$$b_0 = -\Delta R_{p-4}^2 \left(4\beta + 3\gamma^2\right)/B_3, \tag{5}$$

$$b_2 = -\left(2\beta - 3\gamma^2 - 6\right)/B_3, \tag{6}$$

$$B_3 = 10\beta - 12\gamma^2 - 18, \tag{7}$$

$$\gamma = \mu_3/\Delta R_{p-4}^3 \tag{8}$$

represents the skewness,

$$\beta = \mu_4/\Delta R_{p-4}^4 \tag{9}$$

represents the abruptness, and only if the skewness γ and the abruptness β satisfy the following two conditions respectively:

$$\left.\begin{array}{l} 0<\gamma^2<32 \\ \beta > \dfrac{39\gamma^2 + 6(\gamma^2+4)^{3/2} + 48}{32-\gamma^2} \end{array}\right\}. \tag{10}$$

Thus the probability density function of Pearson-IV distribution, which was derived from [3], is given as follows [4,5]:

$$h(z_R) = C\left[-\left(b_2 z_R^2 + b_1 z_R + b_0\right)\right]^{\frac{1}{2b_2}} \cdot \exp\left[-\frac{2a_4}{\sqrt{-4b_2}}\arctan\left(\frac{2b_2 z_R + b_1}{\sqrt{-4b_2}}\right)\right] \text{ or}$$
$$h(z) = k\left[1 + \left(\frac{z-\lambda}{a}\right)^2\right]^{-m} \cdot \exp\left[-v \arctan\left(\frac{z-\lambda}{a}\right)\right], \tag{11}$$

where, C and k are corresponding normalization constants respectively,

$$a = \pm\sqrt{-\frac{1}{b_2}}, \tag{12}$$

$$\lambda = -\frac{b_1}{2b_2}, \tag{13}$$

$$m = -\frac{1}{2b_2}, \tag{14}$$

$$v = \frac{2a_4}{\sqrt{-4b_2}}, \tag{15}$$

are all real-valued parameters with relatively obvious physical meanings and much easier to use in ion implantation field for $m>0.5$ and $b_0 = b_1^2/4b_2 - 1$.

Therefore the moments are given as follows based on [5,6] with $r = 2(m-1)$:

$$R_{p-4} = \begin{cases} \lambda - \dfrac{av}{2(m-1)} & (m>1) \\ -\infty & (v>0, m\le 1) \\ +\infty & (v<0, m\le 1) \end{cases}, \tag{16}$$

$$\Delta R_{p-4} = \left|\frac{a}{r}\right|\sqrt{\frac{(r^2+v^2)}{(r-1)}} \quad (m>1.5), \tag{17}$$

$$\mu_3 = -\left(\frac{a}{r}\right)^3 \frac{4v(r^2+v^2)}{(r-1)(r-2)} \quad (m>2), \tag{18}$$

$$\mu_4 = \left(\frac{a}{r}\right)^4 \frac{3(r^2+v^2)[(r+6)(r^2+v^2) - 8r^2]}{(r-1)(r-2)(r-3)} \quad (m>2.5). \tag{19}$$

Substitute the Eqs. (17)–(19) into Eqs. (8) and (9) respectively we can obtain the expressions for the skewness and the abruptness:

$$\gamma = \mp\frac{4v}{r-2}\sqrt{\frac{r-1}{r^2+v^2}} \quad \left\{\begin{array}{l} a>0 \\ a<0 \end{array}, \quad m>1.5\right\}, \tag{20}$$

$$\beta = \frac{3(r-1)}{(r-2)(r-3)}\left[(r+6) - \frac{8r^2}{r^2+v^2}\right] \quad (m>2.5). \tag{21}$$

The first derivative of the Pearson-IV p.d.f. with respect to z is given by

$$g(z) = \frac{1}{h(z)}\frac{dh}{dz} = -2ma\frac{z - \lambda + \frac{av}{2m}}{a^2 + (z-\lambda)^2}, \tag{22}$$

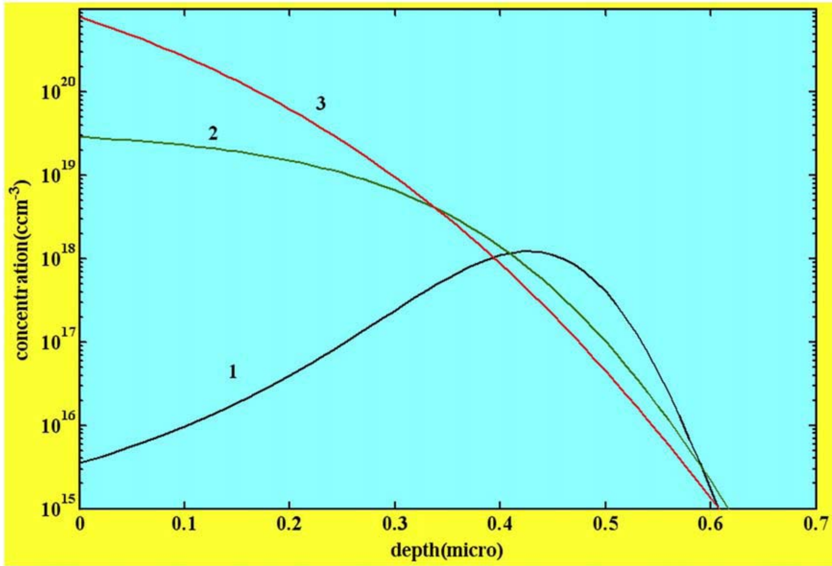

Fig. 2. A group of non-normalized p.d.f.s of Pearson-IV distribution.

which means that there is always a single mode M determined by

$$M = \lambda - \frac{av}{2m}, \tag{23}$$

where the first derivative is zero. The second derivative of $h(z)$ respect to z is given by

$$\frac{1}{h(z)}\frac{d^2h}{dz} = \frac{2ma(2ma+1)\left(z-\lambda+\frac{av}{2m}\right)^2 - 4a^2v\left(z-\lambda+\frac{av}{2m}\right) + \frac{a^3}{2m}(3v^2-4m^2)}{\left[a^2+(z-\lambda)^2\right]^2}, \tag{24}$$

which is zero at exactly two inflection points $\Lambda_{\pm}$ given by

$$\Lambda_{\pm} = M + \frac{a}{m(2ma+1)}\left[v + \sqrt{(2ma+1)(4m^2-3v^2)+v^2}\right]. \tag{25}$$

Eq. (25) indicates that the inflection points are in-equidistant from the mode as long as $v \neq 0$, which is not the same as that in [5]. In another words, the peak profile of the Pearson-IV p.d.f. is not symmetric respective to $z=M$. Thus the setover of distances between the inflection points and the mode is given by

$$\sigma_{\Lambda} = (\Lambda_{+} - M) - (M - \Lambda_{-}) = \frac{2av}{m(2m+1)}. \tag{26}$$

Eqs. (16), (23), (25) and (26) indicate that the projected distance of Pearson-IV distribution R_{p-4} decreases with v and the setover of distances between the inflection points and the mode is proportional to v at the same time for $m>1$, which means the abruptness of the front semi-peak is higher than that of the back semi-peak respective to $z=M$ as $v<0$ and $R_{p-4}<\lambda$ while lower as $v=0$ and $R_{p-4}>\lambda$.

In order to evaluate Pearson-IV distribution analytically and simply, the normalization constant κ is adopted from [4] as:

$$\kappa = \frac{\gamma^2(\beta+3)^2}{4(2\beta-3\gamma^2-6)(4\beta-3\gamma^2)}, \tag{27}$$

and a new parameter — non-normalization impurity concentration constant N_{p-4} is defined as:

$$N_{p-4} \cong \frac{D_{p-4}}{\sigma_{\Lambda}} = \frac{D_{p-4}m(2m+1)}{2av}. \tag{28}$$

Eq. (28) means N_{p-4} is the p.d.f. of quasi-homogenous distribution approximation in the close interval of $[\Lambda_{-},\Lambda_{+}]$. Thus the non-normalization Pearson-IV distribution can be directly expressed as:

$$H(z) = N_{p-4}\kappa\left[1+\left(\frac{z-\lambda}{a}\right)^4\right]^{-m} \cdot \exp\left[-v\arctan\left(\frac{z-\lambda}{a}\right)\right]. \tag{29}$$

A group of non-normalized p.d.f.s of Pearson-IV distribution are illustrated in Fig. 2 with $m=10$, $\lambda=0.5$, $D_{p-4}=5\text{e}11\ \text{cm}^{-2}$

Table 1
Some main parameters for the group of non-normalized p.d.f.s of Pearson-IV distribution

Number	M	a	v	$N_{p-4}\kappa$	σ_{Λ}
1	0.4350	0.2000	6.500	4.0385e17	0.0124
2	0.2400	0.4000	13.00	1.0096e17	0.0495
3	−0.085	0.6000	19.50	4.4870e16	0.1114

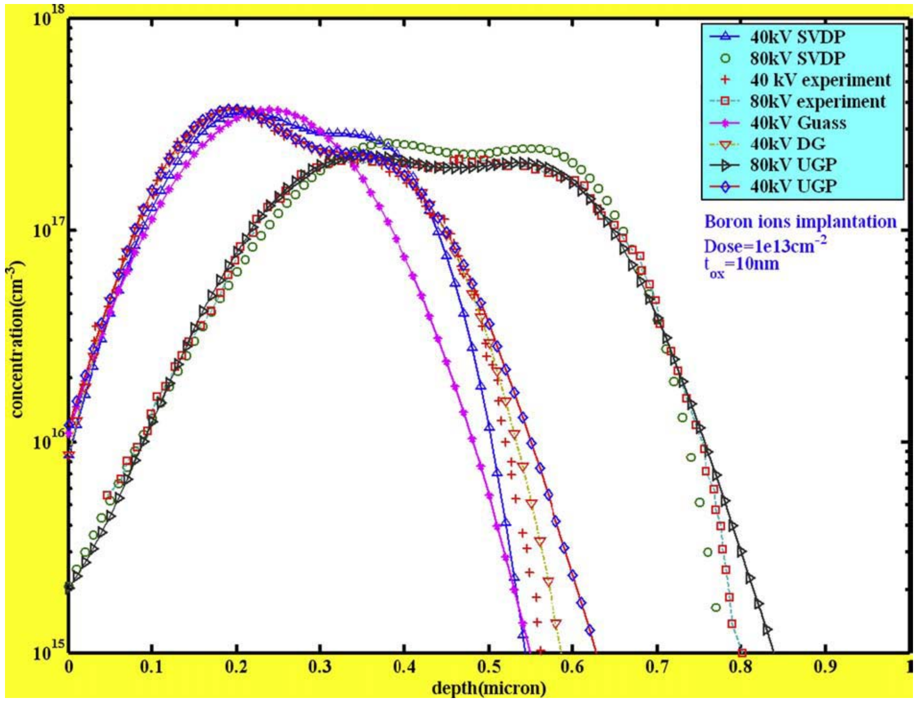

Fig. 3. Results of boron ions implantation.

and other main parameters listed in Table 1. From Fig. 2 it can be seen that Pearson-IV distribution is very suitable for channeling component description in different ions implantation cases.

2.3. United Gauss–Pearson-IV distribution

Otherwise, it is better to picture the profile of implanted impurity atoms more precisely for any dumb bell type distributions by adopting a united non-normalization Gauss–Pearson-IV distribution as follows:

$$G(z) = \left(1-R_{\mathrm{g}}\right)Nf(z) + R_{\mathrm{g}}H(z) = \frac{D_g Nf(z) + D_{p-4}H(z)}{D_g + D_{p-4}}. \tag{30}$$

where, $(1-R_g)Nf(R_p)$ represents the peak impurity concentration value of Gauss distribution component for random scattering effect of ion implantation into silicon, $R_gH(M)$ represents the peak impurity concentration value of Pearson-IV distribution component for channeling effect of ion implantation, N is an impurity concentration coefficient for random scattering effect of ion implantation into silicon, M is the mode point mentioned above, and the ratio between the total channeling ion dose implanted into silicon and the total ion dose implanted into silicon, R_g, is defined as:

$$R_{\mathrm{g}} = \frac{D_{p-4}}{D_g + D_{p-4}}, \tag{31}$$

where, D_{p-4} represents the total dose of impurity ions implanted into silicon through the channels of silicon lattice which could be described by Pearson-IV distribution more precisely, D_g represents the total dose of impurity ions implanted into silicon according to random scattering mechanism which could be described by Gauss distribution accurately.

3. Results and discussion

A group of simulated and experimental results of boron ions implantation into Si at the dose of 1e13 cm^{-2}, 10 nm-thick oxide mask, the tilt of 0°, the rotation angle of 0°, the energies of 40 kV and 80 kV are illustrated in Fig. 3 respectively with $m=5$ and other main parameters listed in Table 2. Partial results are adopted from reference [6] in Fig. 3. From Fig. 3 it can be seen that this dumbbell type distribution can be described better with

Table 2
Most of main parameters for the boron ions implantation into Si with 10 nm-thick oxide mask

Number	M	a	v	λ	$N_{p-4\mathrm{K}}$	R_{g}	D	D_{ox}
1	0.357	0.2369	2.25	0.4103	1.65e17	0.2062	1.2195(e12)	8.4637e12
2	0.555	0.2526	2.4	0.6156	1.44e17	0.1818	1.4368e12	8.2439e12

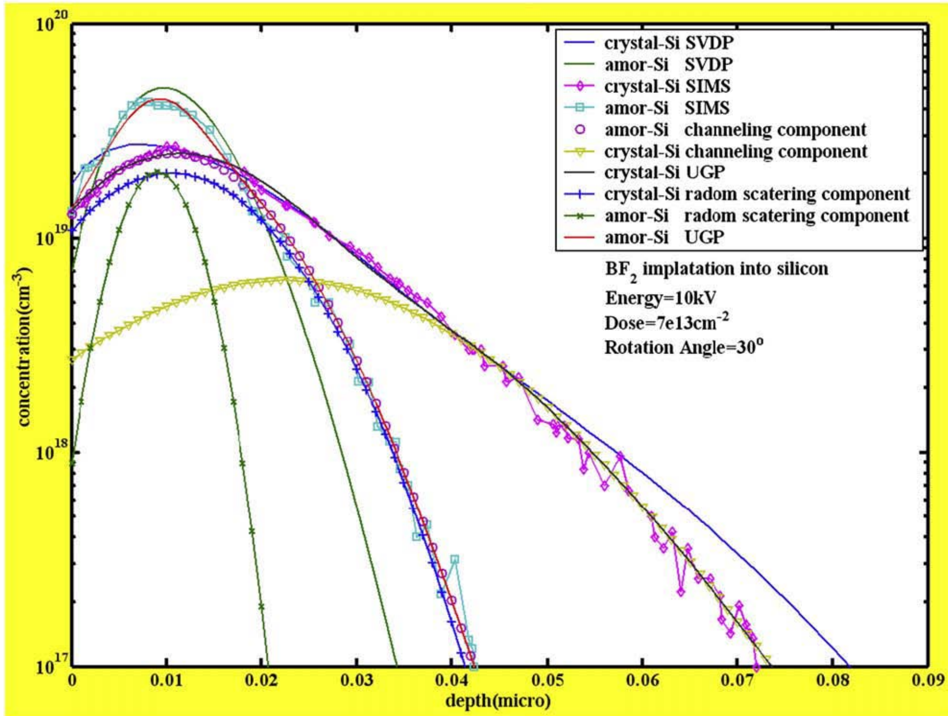

Fig. 4. A group of simulated and experimental results of BF_2 implantation into crystal and amorphous Si.

the UGP distribution model in higher impurity concentration areas while worse in lower impurity concentration areas than with SVDP model. In addition, a Gauss approximation and a double Gauss (DG) approximation for the former results are also illustrated in Fig. 3, which indicates that the DG model might also be suitable for the description of this dumbbell type distribution.

A group of simulated and experimental results of BF_2 implantation into crystal and amorphous Si at the dose of 7e13 cm^{-2}, the tilt of 0◦, the rotation angle of 30◦, the energy of 10 kV are illustrated in Fig. 4 respectively with other main parameters listed in Table 3. The experimental results are adopted from reference [7] and the SVDP results are obtained with Silvaco TCAD ATHENA in Fig. 4. From Fig. 4 it can be seen that both the UGP models fit with the SIMS experimental results very well almost in full region.

4. Conclusions

The analyses and discussions above indicate that the presented UGP model might be very suitable for description of the distributions of ions implanted into silicon with both random scattering and channeling effects, which are widely used in silicon CMOS VLSI technologies, SOI power devices and power integrated circuits technologies.

Acknowledgements

Firstly I acknowledge the TCAD software support provided by Silvaco Inc. Secondly I acknowledge Mr. Lifei Jiang and Wenjie Xu for their help in providing TCAD related data.

References

[1] A.F. Tasch, S.H. Yang, S.J. Morris, Nuclear Instruments and Methods in Physics Research B102 (1995) 173.
[2] K.M. Klein, C. Park, S.H. Yang, et al., IEDM-91, 1991, p. 697.
[3] Y. Nagahara, Statistics & Probability Letters 43 (1999) 251.
[4] Y. Nagahara, Computational Statistics & Data Analysis 47 (2004) 1.
[5] J. Heinrich. A Guide to the Pearson Type IV Distribution. CDF/MEMO/STATISTICS/PUBLIC/6820 (DEC. 21, 2004) 1−12.
[6] Silvaco. Aniiex04.in. The 4th Example of TCAD Ion Implantation Simulation and Verification. Silvaco Inc. (2004) 1
[7] K.B. Parab, M.S. Morris, S.-H. Yang, et al., Proceedings of the 11th IC IT, Piscataway, NJ, USA, 1997, p. 16.

Table 3
Most of main parameters for the BF_2 implantation into crystal and amorphous Si

Item	m	M	a	v	λ	$N_{p-4\kappa}$	R_g
Crystal Si	8.4	0.0224	−0.066667	−1.0	0.0264	6.1677e18	2.357143e−3
Amor-Si	18	0.0105	0.054054	1.0	0.0120	2.4642e19	7.142857e−s3

Please cite this article as: H. Zhang, et al., Solid State Ionics (2008), doi:10.1016/j.ssi.2008.01.084