FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. 28, No 1 (2013), 17–26

ON THE GROWTH OF SOLUTIONS TO NON-HOMOGENOUS LINEAR DIFFERENTIAL EQUATIONS WITH ENTIRE COEFFICIENTS HAVING THE SAME ORDER *

Benharrat Belaïdi and Habib Habib

Abstract. In this paper, we investigate the order of solutions to the non-homogeneous linear differential equation

$$f^{(k)} + B_{k-1} f^{(k-1)} + \cdots + B_l f^{(l)} + \cdots + B_1 f' + B_0 f = F,$$

where $k \geqslant 2$, $B_j(z)$ $(j = 0, 1, \ldots, k-1)$ and $F(z)$ are entire functions of finite order.

Keywords: Differential equation, characteristic function, meromorphic function, complex number.

1. Introduction and statement of results

Throughout this paper, we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna's value distribution theory [5, 9]. In what follows, we give the necessary notations and basic definitions.

Definition 1.1. (see [5, 9]) Let f be a meromorphic function. Then the order $\rho(f)$ of $f(z)$ is defined by

$$\rho(f) = \limsup_{r \longrightarrow +\infty} \frac{\log T(r, f)}{\log r},$$

where $T(r, f)$ is the Nevanlinna characteristic function of f. If f is an entire function, then the order $\rho(f)$ of $f(z)$ is defined by

$$\rho(f) = \limsup_{r \longrightarrow +\infty} \frac{\log T(r, f)}{\log r} = \limsup_{r \longrightarrow +\infty} \frac{\log \log M(r, f)}{\log r},$$

where $M(r, f) = \max_{|z|=r} |f(z)|$.

Received August 4, 2012.; Accepted February 05, 2013.
2010 *Mathematics Subject Classification.* Primary 34M10; Secondary 30D35
*This research is supported by ATRST (Agence Thématique de Recherche en Sciences et Technologies) and University of Mostaganem (UMAB), (PNR Project Code 8/u27/3144).

Definition 1.2. (See [5, 9]) Let f be a meromorphic function. Then the exponent of convergence of the sequence of zeros of $f(z)$ is defined by

$$\lambda(f) = \lim_{r \longrightarrow +\infty} \sup \frac{\log N\left(r, \frac{1}{f}\right)}{\log r},$$

where $N\left(r, \frac{1}{f}\right)$ is the counting function of zeros of $f(z)$ in $\{z : |z| < r\}$. Similarly, the exponent of convergence of the sequence of distinct zeros of $f(z)$ is defined by

$$\overline{\lambda}(f) = \lim_{r \longrightarrow +\infty} \sup \frac{\log \overline{N}\left(r, \frac{1}{f}\right)}{\log r},$$

where $\overline{N}\left(r, \frac{1}{f}\right)$ is the counting function of distinct zeros of $f(z)$ in $\{z : |z| < r\}$.

In [8], Wang and Laine have investigated the growth of solutions to higher order non-homogeneous linear differential equations and obtained the following result.

Theorem 1.1. (See [8])*Suppose that*

$$A_j(z) = h_j(z) e^{P_j(z)}, \ (j = 0, \cdots, k-1),$$

where

$$P_j(z) = a_{j,n} z^n + \cdots + a_{j,0}, \ (j = 0, 1, \cdots, k-1)$$

are polynomials with degree $n \geqslant 1$,

$$h_j(z) \ (\not\equiv 0), \ (j = 0, 1, \ldots, k-1)$$

are entire functions with order less than n, *and that* $H(z) \not\equiv 0$ *is an entire function of order less than* n. *If* $a_{j,n}$ $(j = 0, 1, \cdots, k-1)$ *are distinct complex numbers, then every solution* f *of the differential equation*

$$f^{(k)} + A_{k-1}(z) f^{(k-1)} + \cdots + A_1(z) f' + A_0(z) f = H(z)$$

is of infinite order.

In [7], Peng and Chen have investigated the order and hyper-order of solutions to some second order linear differential equations and have proved the following result.

Theorem 1.2. *Let* $A_j(z)$ $(\not\equiv 0)$ $(j = 1, 2)$ *be entire functions with* $\rho\left(A_j\right) < 1$, a_1, a_2 *be complex numbers such that* $a_1 a_2 \neq 0$, $a_1 \neq a_2$ *(suppose that* $|a_1| \leqslant |a_2|$*). If* $\arg a_1 \neq \pi$ *or* $a_1 < -1$, *then every solution* $f \not\equiv 0$ *of the differential equation*

$$f'' + e^{-z} f' + \left(A_1 e^{a_1 z} + A_2 e^{a_2 z}\right) f = 0$$

is of infinite order and hyper-order

$$\rho_2(f) = \lim_{r \longrightarrow +\infty} \sup \frac{\log \log T(r, f)}{\log r} = 1.$$

The main purpose of this paper is to extend and improve Theorems 1.1-1.2 to some higher order linear differential equations. In fact we will prove the following results.

Theorem 1.3. *Let $k \geqslant 2$ be an integer, $I_j \subset \mathbb{N}$ $(j = 0, 1, \cdots, k-1)$ be finite sets such that $I_j \cap I_m = \varnothing$ $(j \neq m)$ and $I = \cup_{j=0}^{k-1} I_j$. Suppose that*

$$B_j = \sum_{i \in I_j} A_i e^{P_i(z)}, \ (j = 0, 1, \cdots, k-1)$$

where $A_i(z) (\not\equiv 0), (i \in I)$ are entire functions with

$$\max\{\rho(A_i), i \in I\} < n, \ \ P_i(z) = a_{in} z^n + \cdots + a_{i1} z + a_{i0}, \ (i \in I)$$

are polynomials with degree $n \geqslant 1$ and that $F(z) \not\equiv 0$ is an entire function with $\rho(F) < n$. If a_{in} $(i \in I)$ are distinct complex numbers, then every solution f of the differential equation

$$f^{(k)} + B_{k-1} f^{(k-1)} + \cdots + B_l f^{(l)} + \cdots + B_1 f' + B_0 f = F \tag{1.1}$$

satisfies $\rho(f) = +\infty$.

Theorem 1.4. *Under the hypotheses of Theorem 1.3, suppose further that $\varphi(z) \not\equiv 0$ is an entire, then every solution $f \not\equiv 0$ of (1.1) satisfies*

$$\overline{\lambda}(f - \varphi) = \lambda(f - \varphi) = \rho(f) = +\infty.$$

2. Preliminary lemmas

Lemma 2.1. (See [3]) *Let $P_1, P_2, \cdots, P_n$ $(n \geqslant 1)$ be non-constant polynomials with degree $d_1, d_2, \cdots, d_n$, respectively, such that $\deg\left(P_i - P_j\right) = \max\left\{d_i, d_j\right\}$ for $i \neq j$. Let*

$$A(z) = \sum_{j=1}^{n} B_j(z) e^{P_j(z)}$$

where $B_j(z) (\not\equiv 0)$ are entire functions with $\rho\left(B_j\right) < d_j$. Then

$$\rho(A) = \max_{1 \leqslant j \leqslant n}\{d_j\}.$$

Lemma 2.2. (See [2]) *Suppose that $P(z) = (\alpha + i\beta) z^n + \cdots$ (α, β are real numbers, $|\alpha| + \left|\beta\right| \neq 0$) is a polynomial with degree $n \geqslant 1$, that $A(z) (\not\equiv 0)$ is an entire function with $\rho(A) < n$. Set*

$$g(z) = A(z) e^{P(z)}, \ z = re^{i\theta}, \ \delta(P, \theta) = \alpha \cos n\theta - \beta \sin n\theta.$$

Then for any given $\varepsilon > 0$, *there is a set* $E_1 \subset [0, 2\pi)$ *that has linear measure zero, such that for any*

$$\theta \in [0, 2\pi) \setminus (E_1 \cup E_2),$$

there is $R > 0$, *such that for* $|z| = r > R$, *we have*

(*i*) *if* $\delta(P, \theta) > 0$, *then*

$$\exp\{(1-\varepsilon)\delta(P,\theta) r^n\} \leqslant \left|g\left(re^{i\theta}\right)\right| \leqslant \exp\{(1+\varepsilon)\delta(P,\theta) r^n\}; \tag{2.1}$$

(*ii*) *if* $\delta(P, \theta) < 0$, *then*

$$\exp\{(1+\varepsilon)\delta(P,\theta) r^n\} \leqslant \left|g\left(re^{i\theta}\right)\right| \leqslant \exp\{(1-\varepsilon)\delta(P,\theta) r^n\}, \tag{2.2}$$

where $E_2 = \{\theta \in [0, 2\pi) : \delta(P, \theta) = 0\}$ *is a finite set.*

Lemma 2.3. (See [4]) *Let* f *be a transcendental meromorphic function of finite order* ρ. *Let* $\varepsilon > 0$ *be a constant,* k *and* j *be integers satisfying* $k > j \geqslant 0$. *Then the following two statements hold:*

(i) There exists a set $E_3 \subset (1, +\infty)$ *which has finite logarithmic measure, such that for all* z *satisfying* $|z| \notin E_3 \cup [0, 1]$, *we have*

$$\left|\frac{f^{(k)}(z)}{f^{(j)}(z)}\right| \leqslant |z|^{(k-j)(\rho-1+\varepsilon)}.$$

(ii) There exists a set $E_4 \subset [0, 2\pi)$ *which has linear measure zero, such that if* $\theta \in [0, 2\pi) \backslash E_4$, *then there is a constant* $R = R(\theta) > 0$ *such that* (2.3) *holds for all* z *satisfying* $\arg z = \theta$ *and* $|z| \geqslant R$.

Lemma 2.4. (See [8])*Let* $f(z)$ *be an entire function and suppose that*

$$G(z) := \frac{\log^+ \left|f^{(k)}(z)\right|}{|z|^\rho}$$

is unbounded on some ray $\arg z = \theta$ *with constant* $\rho > 0$. *Then there exists an infinite sequence of points*

$$z_n = r_n e^{i\theta}, \ (n = 1, 2, \cdots),$$

where $r_n \to +\infty$, *such that* $G(z_n) \to \infty$ *and*

$$\left|\frac{f^{(j)}(z_n)}{f^{(k)}(z_n)}\right| \leqslant \frac{1}{(k-j)!}(1+o(1))\, r_n^{k-j}, j = 0, 1, \cdots, k-1$$

as $n \to +\infty$.

Lemma 2.5. (See [8]) *Let $f(z)$ be an entire function with $\rho(f) = \rho < +\infty$. Suppose that there exists a set $E_5 \subset [0, 2\pi)$ which has linear measure zero, such that $\log^+ \left|f\left(re^{i\theta}\right)\right| \leqslant Mr^\sigma$ for any ray $\arg z = \theta \in [0, 2\pi) \setminus E_5$, where M is a positive constant depending on θ, while σ is a positive constant independent of θ. Then $\rho(f) \leqslant \sigma$.*

Lemma 2.6. (See [1]) *Let*

$$A_j\,(j = 0, 1, \cdots, k-1),\; F \not\equiv 0$$

be finite order meromorphic functions. If $f(z)$ is an infinite order meromorphic solution of the differential equation

$$f^{(k)} + A_{k-1} f^{(k-1)} + \cdots + A_1 f' + A_0 f = F,$$

then f satisfies

$$\overline{\lambda}(f) = \lambda(f) = \rho(f) = \infty.$$

3. Proof of Theorem 1.3

Proof. First we prove that every solution of (1.1) satisfies $\rho(f) \geqslant n$. We assume that $\rho(f) < n$. Rewrite (1.1) as

$$\sum_{i\in I_{k-1}} A_i f^{(k-1)} e^{P_i(z)} + \cdots + \sum_{i\in I_1} A_i f' e^{P_i(z)} + \sum_{i\in I_0} A_i f e^{P_i(z)} = F - f^{(k)}. \tag{3.1}$$

Since a_{in} $(i \in I)$ are distinct complex numbers, then by (3.1) and the Lemma 2.1, we have

$$n = \rho\left\{\sum_{i\in I_{k-1}} A_i f^{(k-1)} e^{P_i(z)} + \cdots + \sum_{i\in I_0} A_i f e^{P_i(z)}\right\} = \rho\left\{F - f^{(k)}\right\} < n.$$

This is a contradiction. Hence, $\rho(f) \geqslant n$. Therefore f is a transcendental solution of equation (1.1).

Now we prove that $\rho(f) = +\infty$. Suppose that $\rho(f) = \rho < +\infty$. Since $\rho(F) < n$, then for any given ε $(0 < 2\varepsilon < \min\{1, n - \rho(F)\})$ and for sufficiently large r, we have

$$|F(z)| \leqslant \exp\left\{r^{\rho(F)+\varepsilon}\right\}. \tag{3.2}$$

By Lemma 2.2, there exists a set $E \subset [0, 2\pi)$ of linear measure zero, such that whenever $\theta \in [0, 2\pi) \setminus E$, then $\delta(P_i, \theta) \neq 0$ for all $i \in I$ and $\delta(P_i, \theta) \neq \delta(P_m, \theta)$ for all i, m with $m < i$ $(i, m \in I)$. If $z = re^{i\theta}$ has r large enough, then each $A_i(z)e^{P_i(z)}$ satisfies either (2.1) or (2.2). By Lemma 2.3, there exists a set $E_4 \subset [0, 2\pi)$ which has linear measure zero, such that if $\theta \in [0, 2\pi) \setminus E_4$, then there is a constant $R = R(\theta) > 1$ such that for all z satisfying $\arg z = \theta$ and $|z| \geqslant R$, we have

$$\left|\frac{f^{(j)}(z)}{f^{(i)}(z)}\right| \leqslant |z|^{k\rho}, 0 \leqslant i < j \leqslant k. \tag{3.3}$$

Since a_{in} $(i \in I)$ are distinct complex numbers, then for any fixed

$$\theta \in [0, 2\pi) \setminus (E \cup E_4),$$

there exists exactly one $s \in I$ such that

$$\delta(P_s, \theta) = \delta = \max\{\delta(P_i, \theta), i \in I\}$$

and there exists $l \in \{0, 1, \cdots, k-1\}$ such that $s \in I_l$. Set

$$\delta_1 = \max\{\delta(P_i, \theta) : i \neq s,\ i \in I\},$$

then $\delta_1 < \delta$ and $\delta \neq 0$. We now discuss two cases separately.

Case 1: Suppose that $\delta > 0$. By Lemma 2.2, for any given ε with

$$0 < 2\varepsilon < \min\left\{\frac{\delta - \delta_1}{\delta}, n - \rho(F)\right\},$$

we obtain

$$\left|A_s(z)e^{P_s(z)}\right| \geqslant \exp\{(1-\varepsilon)\delta r^n\}, s \in I_l, \tag{3.4}$$

$$\left|A_i(z)e^{P_i(z)}\right| \leqslant \exp\{(1+\varepsilon)\delta_1 r^n\} \tag{3.5}$$

for $i \neq s$ and for sufficiently large r. We now prove that

$$\log^+ \left|f^{(l)}(z)\right| / |z|^{\rho(F)+\varepsilon}$$

is bounded on the ray $\arg z = \theta$. We assume that

$$\log^+ \left|f^{(l)}(z)\right| / |z|^{\rho(F)+\varepsilon}$$

is unbounded on the ray $\arg z = \theta$. Then by Lemma 2.4, there is a sequence of points $z_m = r_m e^{i\theta}$, such that $r_m \to +\infty$, and that

$$\frac{\log^+ \left|f^{(l)}(z_m)\right|}{r_m^{\rho(F)+\varepsilon}} \to +\infty, \tag{3.6}$$

$$\left|\frac{f^{(j)}(z_m)}{f^{(l)}(z_m)}\right| \leqslant \frac{1}{(l-j)!}(1+o(1))\, r_m^{l-j}, (j = 0, \cdots, l-1). \tag{3.7}$$

From (3.2) and (3.6), we get

$$\left|\frac{F(z_m)}{f^{(l)}(z_m)}\right| \to 0 \tag{3.8}$$

for m is large enough. From (1.1), we obtain

$$
\begin{aligned}
\left|A_s e^{P_s(z_m)}\right| \leqslant\ & \left|\frac{f^{(k)}(z_m)}{f^{(l)}(z_m)}\right| + \left|\sum_{i\in I_{k-1}} A_i e^{P_i(z_m)}\right| \left|\frac{f^{(k-1)}(z_m)}{f^{(l)}(z_m)}\right| \\
& + \cdots + \left|\sum_{i\in I_{l+1}} A_i e^{P_i(z_m)}\right| \left|\frac{f^{(l+1)}(z_m)}{f^{(l)}(z_m)}\right| + \left|\sum_{i\in I_l, i\neq s} A_i e^{P_i(z_m)}\right| \\
& + \left|\sum_{i\in I_{l-1}} A_i e^{P_i(z_m)}\right| \left|\frac{f^{(l-1)}(z_m)}{f^{(l)}(z_m)}\right| + \cdots + \left|\sum_{i\in I_1} A_i e^{P_i(z_m)}\right| \left|\frac{f'(z_m)}{f^{(l)}(z_m)}\right| \\
& + \left|\sum_{i\in I_0} A_i e^{P_i(z_m)}\right| \left|\frac{f(z_m)}{f^{(l)}(z_m)}\right| + \left|\frac{F(z_m)}{f^{(l)}(z_m)}\right|.
\end{aligned} \tag{3.9}
$$

Substituting (3.3), (3.4), (3.5), (3.7) and (3.8) into (3.9), we have

$$\exp\{(1-\varepsilon)\delta r_m^n\} \leqslant M_0 \exp\{(1+\varepsilon)\delta_1 r_m^n\} r_m^{M_1}, \tag{3.10}$$

where $M_0 > 0$ and $M_1 > 0$ are some constants. By $0 < \varepsilon < \frac{\delta-\delta_1}{2\delta}$ and (3.10), we can get

$$\exp\left\{\frac{(\delta-\delta_1)^2}{2\delta} r_m^n\right\} \leqslant M_0 r_m^{M_1}$$

which is a contradiction. Therefore,

$$\log^+\left|f^{(l)}(z)\right| / |z|^{\rho(F)+\varepsilon}$$

is bounded, and we have

$$\left|f^{(l)}(z)\right| \leqslant M \exp\left\{r^{\rho(F)+\varepsilon}\right\}$$

on the ray $\arg z = \theta$. By the same reasoning as in the proof of Lemma 3.1 in [6], we immediately conclude that

$$
\begin{aligned}
\left|f(z)\right| &\leqslant (1+o(1))\, r^l \left|f^{(l)}(z)\right| \\
&\leqslant (1+o(1))\, M r^l \exp\left\{r^{\rho(F)+\varepsilon}\right\} \leqslant M \exp\left\{r^{\rho(F)+2\varepsilon}\right\}
\end{aligned}
$$

on the ray $\arg z = \theta$.

Case 2: Suppose now that $\delta < 0$. From (1.1), we get

$$-1 = B_{k-1}\frac{f^{(k-1)}}{f^{(k)}} + \cdots + B_1\frac{f'}{f^{(k)}} + B_0\frac{f}{f^{(k)}} - \frac{F}{f^{(k)}}. \tag{3.11}$$

By Lemma 2.2, for any given ε with

$$0 < 2\varepsilon < \min\{1, n-\rho(F)\},$$

we have

$$\left|A_i(z)e^{P_i(z)}\right| \leqslant \exp\{(1-\varepsilon)\delta r^n\}, i \in I \tag{3.12}$$

for sufficiently large r. We now prove that

$$\log^+\left|f^{(k)}(z)\right| / |z|^{\rho(F)+\varepsilon}$$

is bounded on the ray $\arg z = \theta$. We assume that

$$\log^+\left|f^{(k)}(z)\right| / |z|^{\rho(F)+\varepsilon}$$

is unbounded on the ray $\arg z = \theta$. Then by Lemma 2.4 there is a sequence of points $z_m = r_m e^{i\theta}$, such that $r_m \to +\infty$, and that

$$\frac{\log^+\left|f^{(k)}(z_m)\right|}{r_m^{\rho(F)+\varepsilon}} \to +\infty, \tag{3.13}$$

$$\left|\frac{f^{(j)}(z_m)}{f^{(k)}(z_m)}\right| \leqslant \frac{1}{(k-j)!}(1+o(1))r_m^{k-j}, (j=0,\ldots,k-1). \tag{3.14}$$

From (3.2) and (3.13), we get

$$\left|\frac{F(z_m)}{f^{(k)}(z_m)}\right| \to 0 \tag{3.15}$$

for m is large enough. Substituting (3.12), (3.14) and (3.15) into (3.11), we get

$$\begin{aligned} 1 \leqslant & \left|\sum_{i\in I_{k-1}} A_i e^{P_i(z_m)}\right| \left|\frac{f^{(k-1)}(z_m)}{f^{(k)}(z_m)}\right| + \cdots + \left|\sum_{i\in I_1} A_i e^{P_i(z_m)}\right| \left|\frac{f'(z_m)}{f^{(k)}(z_m)}\right| \\ & + \left|\sum_{i\in I_0} A_i e^{P_i(z_m)}\right| \left|\frac{f(z_m)}{f^{(k)}(z_m)}\right| + \left|\frac{F(z_m)}{f^{(k)}(z_m)}\right| \leqslant M_2 \exp\{(1-\varepsilon)\delta r_m^n\} r_m^{M_3}, \end{aligned} \tag{3.16}$$

where $M_2 > 0$ and $M_3 > 0$ are some constants. By $\delta < 0$, we have

$$M_2 \exp\{(1-\varepsilon)\delta r_m^n\} r_m^{M_3} \to 0$$

as $r_m \to +\infty$. From (3.16), we get $1 \leqslant 0$ as $r_m \to +\infty$, which is a contradiction. Hence, we have $\left|f^{(k)}(z)\right| \leqslant M\exp\left\{r^{\rho(F)+\varepsilon}\right\}$ on the ray $\arg z = \theta$. This implies, as in Case 1, that

$$\left|f(z)\right| \leqslant M\exp\left\{r^{\rho(F)+2\varepsilon}\right\}. \tag{3.17}$$

Therefore, for any given $\theta \in [0,2\pi)\backslash(E \cup E_4)$, we have got (3.17) on the ray $\arg z = \theta$, provided that r is large enough. Then by Lemma 2.5, we have $\rho(f) \leqslant \rho(F) + 2\varepsilon < n$, which is a contradiction. Hence every transcendental solution f of (1.1) must be of infinite order. □

4. Proof of Theorem 1.4

Proof. Suppose that f is a solution of equation (1.1). Then, by Theorem 1.3 we have $\rho(f) = +\infty$. Set

$$g(z) = f(z) - \varphi(z),$$

$g(z)$ is an entire function and

$$\rho(g) = \rho(f) = +\infty.$$

Substituting $f = g + \varphi$ into (1.1), we have

$$g^{(k)} + B_{k-1}g^{(k-1)} + \cdots + B_1 g' + B_0 g = D, \tag{4.1}$$

where

$$D = F - \left[\varphi^{(k)} + B_{k-1}\varphi^{(k-1)} + \cdots + B_1\varphi' + B_0\varphi\right].$$

We prove that $D \not\equiv 0$. In fact, if $D \equiv 0$, then

$$\varphi^{(k)} + B_{k-1}\varphi^{(k-1)} + \cdots + B_1\varphi' + B_0\varphi = F.$$

Hence $\rho(\varphi) = +\infty$, which is a contradiction. Therefore $D \not\equiv 0$. We know that the functions $B_j\ (j = 0, \cdots, k-1)$, D are of finite order. By Lemma 2.6 and (4.1) we have

$$\overline{\lambda}(g) = \lambda(g) = \rho(g) = \rho(f) = +\infty.$$

Therefore

$$\overline{\lambda}(f - \varphi) = \lambda(f - \varphi) = \rho(f) = +\infty,$$

which completes the proof. □

REFERENCES

1. Z. X. Chen: *Zeros of meromorphic solutions of higher order linear differential equations*, Analysis 14 (1994), no. 4, 425-438.
2. Z. X. Chen: *The growth of solutions of* $f'' + e^{-z}f' + Q(z)f = 0$ *where the order* $(Q) = 1$, Sci. China Ser. A 45 (2002), no. 3, 290–300.
3. S. A. Gao, Z. X. Chen, T. W. Chen: *The Complex Oscillation Theory of Linear Differential Equations*, Middle China University of Technology Press, Wuhan, China, 1998. (in Chinese).
4. G. G. Gundersen: *Estimates for the logarithmic derivative of a meromorphic function, plus similar estimates*, J. London Math. Soc. (2) 37 (1988), no. 1, 88–104.
5. W. K. Hayman: *Meromorphic functions*, Oxford Mathematical Monographs Clarendon Press, Oxford 1964.
6. I. Laine, R. Yang: *Finite order solutions of complex linear differential equations*, Electron. J. Diff. Equ., 2004, No. 65, 1-8.
7. F. Peng, Z. X. Chen: *On the growth of solutions of some second-order linear differential equations*, J. Inequal. Appl. 2011, Art. ID 635604, 1–9.

8. J. Wang, I. Laine: *Growth of solutions of nonhomogeneous linear differential equations,* Abstr. Appl. Anal. 2009, Art. ID 363927, 1-11.

9. C. C. Yang, H. X. Yi: *Uniqueness theory of meromorphic functions*, Mathematics and its Applications, 557. Kluwer Academic Publishers Group, Dordrecht, 2003.

Benharrat Belaïdi
Department of Mathematics
Laboratory of Pure and Applied Mathematics
P.O. Box 227
University of Mostaganem (UMAB), Mostaganem, Algeria
belaidi@univ-mosta.dz

Habib Habib
Department of Mathematics
Laboratory of Pure and Applied Mathematics
P.O. Box 227
University of Mostaganem (UMAB), Mostaganem, Algeria
habibhabib2927@yahoo.fr